ISSN 2278-5612

An inventory model for deteriorating items under time varying demand condition.

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ABSTRACT

In this paper we discuss the development of an inventory model for deteriorating items which investigates an instantaneous replenishment model for the items under cost minimization. A time varying type of demand rate with infinite time horizon, exponential deterioration and with shortage in considered. The result is illustrated with numerical example.

.Keywords

Time varying Demand, Optimal control, Inventory system

SUBJECT CLASSIFICATION

AMS Classification No. 90B05

Council for Innovative Research

Peer Review Research Publishing System

Journal: International Journal of Management & Information Technology

Vol. 8, No. 1
editor@cirworld.com
www.cirworld.com, member.cirworld.com

INTRODUCTION

In inventory problems deterioration is defined as damage, spoilage, decay, obsolescence, evaporation, pilferage etc. that result in decrease of usefulness of the original one. It is reasonable to note that a product may be understood to have a lifetime which ends when utility reaches zero. The decrease or loss of utility due to decay is usually a function of the on-hand inventory. For items such as steel, hardware, glassware and toys, the rate of deterioration is so low that there is little need for considering deterioration in the determination of the economic lot size. But some items such as blood, fish, strawberry, alcohol, gasoline, radioactive chemical, medicine and food grains (i.e., paddy, wheat, potato, onion etc.) deteriorate remarkably overtime.

Whitin[17] considered an inventory model for fashion goods deteriorating at the end of a prescribed storage period. Ghare and Scharder [6] developed an EOQ model with an exponential decay and a deterministic demand. Wee[16] developed EOQ models to allow deterioration and an exponential demand pattern. Emmone[5] established a replenishment model for radioactive nuclide generators. The assumption of the constant deterioration rate was relaxed by Covert and Philip[3], who used a two-parameter Weibull distribution to represent the distribution of time to deterioration. This model was further generalized by Philip[11] by taking three-parameter Weibull distribution for deterioration.

Variation in the demand rate plays an important role in the inventory management. Therefore, decisions of inventory are to be made because of the present and future demands. Demand may be constant, time-varying, stock-dependent, price-dependent etc. The constant demand is valid, only when the phase of the product life cycle is matured and also for finite periods of time. Wagner and Whitin [15] discussed the discrete case of the dynamic version of EOQ. EOQ models for deteriorating items with trended demands were considered by Bahari-Kashani [1], Goswami and Chaudhuri[7]. R.P.Tripathi[13] developed a model under time-varying demand rate and holding cost.

In the present competitive market, the effect if marketing policies and conditions such as the price variations and advertisement of an item changes its selling rate amongst the public. In selecting of an item for use, the selling price of an item is one of the decisive factors to the customers. It is commonly seen that lesser selling price causes increases in the selling rate whereas higher selling price has the reverse effect. Hence, the selling rate of an item is dependent on the selling price of that item. This selling rate function must be a decreasing function with respect to the selling price. Incorporating the price variations, recently several researchers i.e. Urban [14], Ladany and Sternleib[9], Subramanyam amd kumaraswamy[12], Goyal and Gunasekaran[8], Bhunia and Maiti[2], Luo[10], and Das et.al [4] developed their models for deteriorating and non-deteriorating items.

In the present chapter, an economic order quantity model is developed for exponential deteriorating items for time varying demand rate. Here the backlogging rate is assumed to be variable and dependent on the waiting time for the next replenishment. The time horizon is classified into two intervals. In the 1st interval the given stock is decreased to zero level due to the combined effect of amelioration, deterioration and demand. In the next interval the shortages are allowed up to the time where some of the shortages are backlogged and rest are lost.

ASSUMPTIONS AND NOTATIONS

Following assumptions are made for the proposed model:

i. Time varying Demand rate is considered.
ii. Single inventory will be used.
iii. Lead time is zero.
iv. Shortages are allowed and partially backlogged with the backlogging rate $\frac{1}{1+\delta(T-t)}$ where the backlogging parameter δ is a non-negative constant.
v. Replenishment rate is infinite but size is finite.
vi. Time horizon is finite.
vii. There is no repair of deteriorated items occurring during the cycle.
viii. Deterioration occurs when the item is effectively in stock.

Following notations are made for the given model:

$I(t)$ = On hand inventory at time t.

$R(t)=\lambda_0 t^{-\beta}$ = Time varying demand rate where $\lambda_0>0$ .and $0<\beta<1.$

ISSN 2278-5612

The distribution of the time to deterioration of an item follows the exponential distribution $\theta(t)$ where $\theta(t)=\begin{cases}\theta\, e^{\theta t}, & for\ t>0\\ 0 \quad , & otherwise\end{cases}$,where θ $(0<\theta<1)$ is called as deterioration rate.

$I(0)$ =Inventory at time $t=0$.

$Q=$ On-hand inventory.

$T=$ Duration of a cycle.

$p_c =$ The purchasing cost per unit item.

$d_c =$ The deterioration cost per unit item.

$o_c =$ The opportunity cost per unit item.

$h_c =$ The holding cost per unit item.

$b_c =$ The shortage cost per unit item.

FORMULATION

The aim of this model is to optimize the total cost incurred and to determine the optimal ordering level. In the interval $[0,t_1]$ the stock will be decreased due to the effect of deterioration and demand. At time t_1, the inventory level reaches zero and in the next interval the shortage are allowed up to time where some of the shortage are backlogged and rest are lost. Only backlogged items are replaced in the next lot.

If $I(t)$ be the on hand inventory at time $t\geq 0$, then at time $t+\Delta t$, the on hand inventory in the interval $[0,t_1]$ will be

$$I(t+\Delta t)=I(t)-\theta(t)\,I(t).\Delta t-\lambda_0\,t^{-\beta}.\Delta t$$

Dividing by Δt and then taking as $\Delta t\to 0$ we get

(1) $$\frac{dI}{dt}=-\theta\,e^{\theta t}I(t)-\lambda_0\,t^{-\beta} \text{ for } 0\leq t\leq t_1.$$

In the end interval , $[t_1,T]$

$$I(t+\Delta t)=I(t)-\frac{\lambda_0\,t^{-\beta}}{1+\delta(T-t)}.\Delta t$$

Dividing by Δt and then taking as $\Delta t\to 0$, we get,

(2) $$\frac{dI}{dt}=-\frac{\lambda_0\,t^{-\beta}}{1+\delta(T-t)},t_1\leq t\leq T.$$

Now solving equation **(1)** with boundary condition $I(t_1)=0$

(3) $$I(t)=\frac{\lambda_0}{2+\theta\,t}\left[\frac{2}{1-\beta}\left\{t_1^{1-\beta}-t^{1-\beta}\right\}+\frac{\theta}{2-\beta}\left\{t_1^{2-\beta}-t^{2-\beta}\right\}\right] \text{ for } 0\leq t\leq t_1.$$

On solving equation **(2)** with boundary condition $I(t_1)=0$

(4) $$I(t)= \lambda_0\left[\frac{(1-\delta T)}{1-\beta}\left\{t_1^{1-\beta}-t^{1-\beta}\right\}+\frac{\delta}{2-\beta}\left\{t_1^{2-\beta}-t^{2-\beta}\right\}\right] \quad \text{for } t_1\le t\le T .$$

Form equation **(3)**, we obtain the initial inventory level.

(5) $$I(0)=\frac{\lambda_0}{2}\left[\frac{2\,t_1^{1-\beta}}{1-\beta}+\frac{\theta}{2-\beta}t_1^{2-\beta}\right].$$

The total inventory holding during the time interval $[0,t_1]$ is given by,

(6) $$I_T=\int_0^{t_1} I\,dt = \frac{\lambda_0}{2}\left[\frac{2\,t_1^{2-\beta}}{2-\beta}+\frac{\theta(1-\theta)}{2(3-\beta)}t_1^{3-\beta}\right]$$

From of equation **(4)** amount of shortage during the time interval $[t_1,T]$ is

(7) $$B_T=\int_{t_1}^{T} -I\,dt$$

$$=-\lambda_0\left[\left\{\frac{(1-\delta T)}{1-\beta}t_1^{1-\beta}+\frac{\delta}{2-\beta}t_1^{2-\beta}\right\}.\{T-t_1\}-\frac{(1-\delta T)}{(1-\beta)(2-\beta)}\left\{T^{2-\beta}-t_1^{2-\beta}\right\}-\frac{\delta}{(2-\beta)(3-\beta)}\left\{T^{3-\beta}-t_1^{3-\beta}\right\}\right]$$

The amount of lost sell during the interval $[t_1,T]$ in given by,

(8) $$L_T=\int_{t_1}^{T} R[1-\frac{1}{1+\delta(T-t)}]dt = \lambda_0\,\delta\left[\frac{T^{2-\beta}}{(1-\beta)(2-\beta)}-\frac{T.t_1^{1-\beta}}{1-\beta}+\frac{t_1^{2-\beta}}{2-\beta}\right].$$

The total number of deteriorated units during the inventory cycle is given by,

(9) $$D_T=\int_0^{t_1}\theta(t)\,I(t)dt$$

$$=\frac{\lambda_0\,\theta}{4}\left[\left\{\frac{2\,t_1^{1-\beta}}{1-\beta}+\frac{\theta\,t_1^{2-\beta}}{2-\beta}\right\}\left\{2\,t_1+\frac{t_1^2}{2}+\frac{t_1^3}{3}\right\}-\frac{4}{(1-\beta)(2-\beta)}t_1^{2-\beta}\right.$$

$$\left.-\frac{2\{\theta(1-\beta)+2-\beta\}}{(1-\beta)(2-\beta)(3-\beta)}t_1^{3-\beta}-\frac{\{\theta(1-\beta)+2(2-\beta)\}}{(1-\beta)(2-\beta)(4-\beta)}t_1^{4-\beta}-\frac{\theta}{(2-\beta)(5-\beta)}t_1^{5-\beta}\right]$$

Using the above equations into consideration the different costs will be as follows.

1. **Purchasing cost per cycle**

(10) $$p_c I(0)=\frac{\lambda_0\,p_c}{2}\left[\frac{2\,t_1^{1-\beta}}{1-\beta}+\frac{\theta}{2-\beta}t_1^{2-\beta}\right]$$

2. **Holding cost per cycle**

(11) $$h_c\int_0^{t_1} I(t)dt = \frac{\lambda_0\,h_c}{2}\left[\frac{2\,t_1^{2-\beta}}{2-\beta}+\frac{\theta(1-\theta)}{2(3-\beta)}t_1^{3-\beta}\right]$$

3. **Deterioration cost per cycle**

$$\textbf{(12)} \quad d_c \int_0^{t_1} \theta(t)\, I(t)dt$$

$$= \frac{\lambda_0\, \theta\, d_c}{4}\left[\left\{\frac{2\, t_1^{1-\beta}}{1-\beta} + \frac{\theta\, t_1^{2-\beta}}{2-\beta}\right\}\left\{2\, t_1 + \frac{t_1^2}{2} + \frac{t_1^3}{3}\right\} - \frac{4}{(1-\beta)(2-\beta)} t_1^{2-\beta}\right.$$

$$\left. - \frac{2\{\theta(1-\beta)+2-\beta\}}{(1-\beta)(2-\beta)(3-\beta)} t_1^{3-\beta} - \frac{\{\theta(1-\beta)+2(2-\beta)\}}{(1-\beta)(2-\beta)(4-\beta)} t_1^{4-\beta} - \frac{\theta}{(2-\beta)(5-\beta)} t_1^{5-\beta}\right]$$

4. **Shortage cost per cycle**

$$\textbf{(13)} \quad -b_c \int_{t_1}^{T} I(t)dt$$

$$= -\lambda_0\, b_c\left[\left\{\frac{(1-\delta\, T)}{1-\beta} t_1^{1-\beta} + \frac{\delta}{2-\beta} t_1^{2-\beta}\right\}.\{T - t_1\} - \frac{(1-\delta\, T)}{(1-\beta)(2-\beta)}\left\{T^{2-\beta} - t_1^{2-\beta}\right\} - \frac{\delta}{(2-\beta)(3-\beta)}\left\{T^{3-\beta} - t_1^{3-\beta}\right\}\right]$$

5. **Opportunity cost due to lost sales per cycle**

$$\textbf{(14)} \quad o_c \int_{t_1}^{T} R\left[1 - \frac{1}{1+\delta(T-t)}\right]dt = \lambda_0\, \delta\, o_c\left[\frac{T^{2-\beta}}{(1-\beta)(2-\beta)} - \frac{T.t_1^{1-\beta}}{1-\beta} + \frac{t_1^{2-\beta}}{2-\beta}\right]$$

The average total cost per unit time of the model will be

$$\textbf{(15)} \quad C(t_1) = \frac{1}{T}\left[\frac{\lambda_0\, p_c}{2}\left[\frac{2\, t_1^{1-\beta}}{1-\beta} + \frac{\theta}{2-\beta} t_1^{2-\beta}\right] + \frac{\lambda_0\, h_c}{2}\left[\frac{2\, t_1^{2-\beta}}{2-\beta} + \frac{\theta(1-\theta)}{2(3-\beta)} t_1^{3-\beta}\right]\right.$$

$$+ \frac{\lambda_0\, \theta\, d_c}{4}\left[\left\{\frac{2\, t_1^{1-\beta}}{1-\beta} + \frac{\theta\, t_1^{2-\beta}}{2-\beta}\right\}\left\{2\, t_1 + \frac{t_1^2}{2} + \frac{t_1^3}{3}\right\} - \frac{4}{(1-\beta)(2-\beta)} t_1^{2-\beta}\right.$$

$$\left. - \frac{2\{\theta(1-\beta)+2-\beta\}}{(1-\beta)(2-\beta)(3-\beta)} t_1^{3-\beta} - \frac{\{\theta(1-\beta)+2(2-\beta)\}}{(1-\beta)(2-\beta)(4-\beta)} t_1^{4-\beta} - \frac{\theta}{(2-\beta)(5-\beta)} t_1^{5-\beta}\right]$$

$$- \lambda_0\, b_c\left[\left\{\frac{(1-\delta\, T)}{1-\beta} t_1^{1-\beta} + \frac{\delta}{2-\beta} t_1^{2-\beta}\right\}.\{T - t_1\} - \frac{(1-\delta\, T)}{(1-\beta)(2-\beta)}\left\{T^{2-\beta} - t_1^{2-\beta}\right\} - \frac{\delta}{(2-\beta)(3-\beta)}\left\{T^{3-\beta} - t_1^{3-\beta}\right\}\right]$$

$$\left. + \lambda_0\, \delta\, o_c\left[\frac{T^{2-\beta}}{(1-\beta)(2-\beta)} - \frac{T.t_1^{1-\beta}}{1-\beta} + \frac{t_1^{2-\beta}}{2-\beta}\right]\right]$$

As it is difficult to solve the problem by deriving a closed equation of the solution of equation (15), Matlab Software has been used to determine optimal t_1^* and hence the optimal $I(0)$, the minimum average total cost per unit time can be determined.

NUMERICAL EXAMPLE

Following example is considered to illustrate the preceding theory.

ISSN 2278-5612

Example

The values of the parameters are considered as follows:

$\theta = 0.2, \delta = 0.1, A = 0.8, T = 1\, Year, \lambda_0 = 200, \beta = 0.7, a_c = \$6/unit, h_c = \$4/unit/year$

$p_c = \$15/unit, d_c = \$9/unit, o_c = \$12/unit, b_c = \$10/unit.$ According to equation (15), we obtain the optimal $t_1^* = 0.5345\, Months$. In addition, the optimal $I^*(0) = 263.425$ units. Moreover, from equation (3.15), we have the minimum average total cost per unit time as $C^* = 260.4\,\$.$

CONCLUSION

Here we have derived an inventory model for deteriorating items. In particular deterioration is considered to be of exponential type. In this model, shortages are allowed and in the shortage period the backlogging rate is variable which depends on the length of the waiting time for the next replenishment. An optimal replenishment policy is derived with minimization of average total cost under the influence of time varying demand. The result is illustrated through numerical example.

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