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Methodology for study of epistemological obstacules: similarity concept

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ABSTRACT

In this document an epistemological analyst, about the concept similarity will be done, by identifying ruptures and affiliations that have been resistant to evolution, the widespread and therefore, could be described as epistemological obstacles. Moreover, the distinction of the three stages of analysis regarding the evolution of the concept of similarity has allowed systematize three approaches to the concept when considered as a teaching object.

Keywords

Epistemological obstacles, historical analysis, similarity, high school

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INTRODUCTION

Nowadays, school books define the “similarity” as an object logically structured and developed. Vision that is a consequence of the unnumbered generalizations that have been made about the concept over the centuries. The previous, entails the necessity of revise the epistemological evolution of the concept and its possible transformations and implications.

One of the purposes of this article is to indicate the epistemic change that suffered the notion of similarity that later on was use for the basis of the geometry development. History gives us an overview of the concept evolution, in which outstands, basically the contributions, made by prominent mathematicians of each period, whom managed to establish the basis of mathematical knowledge. Thus, in a first part, we will analyze the historical development of the concept of “similarity”, making a pause in the most significant problems that have been linked to its evolution path, and show its potential as an articulator of mathematical knowledge through their contextualization in today’s teaching.

In the second part, we have done an epistemological analysis which is a description of the most representative obstacles that have been linked to their evolutionary development.

The first notion of epistemological obstacle that appears for first time in the field of epistemology of experimental science (Bachelard, 1938), was taken and introduced by Brousseau (1967) to the field of mathematics education, getting closer to the causes that lead to the errors “error is not only the effect of ignorance, uncertainty, but is the effect of prior knowledge, which, despite its interest or success, now proves wrong or simply inappropriate" Thus, by making mention of epistemological obstacles, does not necessarily refer to the wrong knowledge, but the kind of knowledge that are hindering the acquisition (construction) of a new one.

This approach will allow us to identify ruptures and affiliations that have been historically resistant to evolution, the widespread and therefore can be described as epistemological obstacles:

The mechanism of the acquisition of knowledge, can be applied to epistemology and history of science, such as learning or teaching. In the one case as in the other, the notion of obstacle is essential to raise the issue of scientific knowledge (Brousseau, 1983, p. 38).

The study of epistemological obstacles its really important in our research, as it will bring an essential knowledge for understanding the determinants of the teaching-learning of the concept of “similarity” in the mathematics course offered in Mexican high schools system. One first point of interest is the identification of the key moments in which the “similarity” had a strong presence, those in which it lived with other mathematical notions, such as proportionality and Theorem Thales; knowledge that influenced his transformation and the realization of concept and its associated symbolic representations.

Regarding the study of epistemological evolution of the concept of similarity, we agree with the theoretical basis of the work done by Lemonidis (1991), who has done historical review of the concept, which relates to the corresponding situation in education, identifying three major periods: Ancient Greece, fifteenth to the eighteenth and nineteenth and twentieth centuries.

CONTRIBUTIONS OF ANCIENT GREECE

The first proof of Theorem of Thales and some other concerning similar figures found in *Euclid's Elements* (s. IV. C.). It is necessary to note in this period the influence of Euclidean geometry to the idea of non-existence of geometric transformations. The notion of similarity has been closely linked with the geometry from its origins. Thales of Miletus, recognized Greek thinker, is considered the first surveyor and one of the seven sages of ancient Greece. Thales is considered the initiator of the deductive method, making a Geometry a rational science, but one which adapts to physical reality in a perfect manner, setting the start for geometric reasoning that would culminate with Euclid. Moreover, by involving the concept similarity in proportional reasoning, concepts of ratio and proportion required.

Historians attribute the Greeks, and especially the Pythagoreans, the development of the theory of proportions, while acknowledging that its origins can be traced back to the Babylonians. Such evidence is provided by a table that is in the British Museum, which refers to several problems related to proportionality (Comin, 2000).

The concept of the Pythagorean School, in tune with the Babylonian thought was based on the idea that "all is number." Thus, for the Pythagoreans numbers could relate to any magnitude. "Although these two concepts were very different, the number corresponding to the arithmetic and number theory, and the magnitude to geometry, however, ruled a desire to unify the number and magnitude in the Pythagorean school" (René de Cotret, 1985, p. 34). They tried to relate numbers and variables using proportions, allowing them to solve geometric problems algebraically. To solve certain quadratic equations, the Pythagoreans used mainly the method of proportions. Now we know that this theory we can find linear segments meeting proportions as:

A) $a:b=c:x$

B) $a:x=x:b$

Where *a*, *b* and *c* are given line segments (Fig 1). By the theorem of proportionality is possible to find the value of *x*, and know the solution A).

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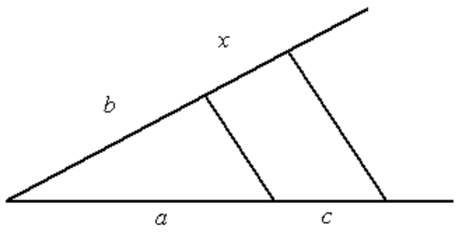

Fig 1

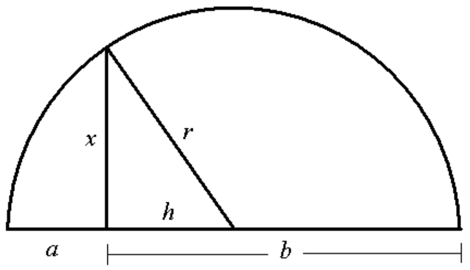

Fig 2

Note that for the last one, what we have to do, is see that $x^2 = ab$ but, through the application of the Pythagorean Theorem, we obtain (Fig 2)

$$x^2 = r^2 - h^2$$

This because of the observation of the past section is the same as

$$x^2 = (r+h)(r-h) = b \cdot \mathrm{a}$$

At first, the above reasoning was applicable mainly to commensurable magnitudes. The discovery of the incommensurability of the diagonal of a square with one side, made by the same Pythagoreans, ended the adequacy world of integers, which helped spread the study of numbers and the magnitudes. This is evident in *Euclid's Elements* (300 B.C.), for arithmetic and magnitudes are working independently: Book VII refers to the proportional numbers, and the book V talk about proportional magnitudes.

In this paper we present a systematization of the studies conducted by the Pythagoreans. Specifically, the study of the proportions, trying to compare two numbers lengths or not through their differences, but by their quotient. One of the first problems encountered was the following:

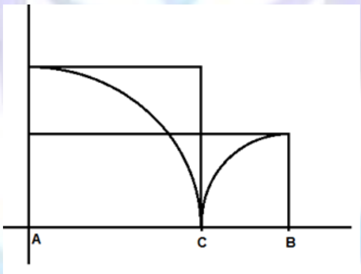

Fig 3

Given a line segment *AB* and divide it into two parts, say *AC* and *CB* such that the area of the square with sides *AC* equals the area of the rectangle with sides *AC* and *CB*. If this problem we took the plane (Fig 3), we know that the solution is.

$$AC = AB\left(\frac{\sqrt{5}\text{-}1}{2}\right)$$

The study of this problem led to the concept of the golden ratio, which is defined as follows: The Golden Mean a M_A quantity is such that the amounts are in golden ratio to M_A this means that their proportion is the golden ratio $\frac{\sqrt[2]{5}-1}{2}$

The geometrical construction of the golden average is calculated as follows:

If we start from a segment *AB* of length $|AB| = x$ and we match the end *A* to the origin and the segment *AB* with the horizontal axis, we can plot the *BC* segment of length $\frac{x}{2}$ perpendicular to segment *AB* and passing through *B*. Thus, we construct the triangle *ABC* we have to point *C* is the Golden Mean quantities *A* and *B*. In geometric terms the approach would be:

AB is a segment of length *x*, can take the midpoint of *AB* and *BC* call it so that the length of *BC* is $\frac{x}{2}$ these two segments as hicks construct the triangle *ABC* (Fig 4).

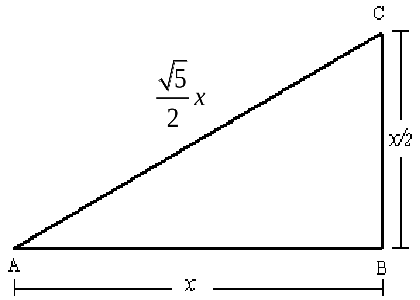

Fig 4

Note that the hypotenuse of this triangle is $\frac{\sqrt{5}}{2}x$ now we draw the circle with center *C* and radius $\frac{x}{2}$ (Fig 5).

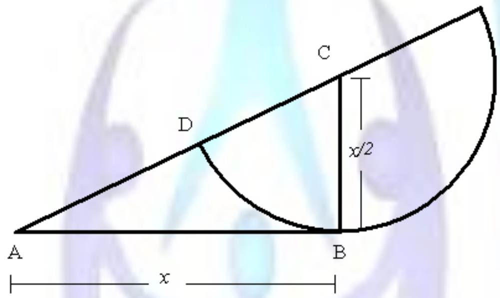

Fig 5

Denote by D the point where the AC segment and girth are cut. Now calculate the length of segment AD as follows:

$$|AD| = \frac{\sqrt[2]{5}}{2}x - \frac{x}{2},$$

$$|AD| = \frac{\sqrt[2]{5}-1}{2}x,$$

or equivalently

$$\frac{|AD|}{x} = \frac{\sqrt[2]{5}-1}{2}$$

That is, the segment *AD* is golden ratio with the length of the segment *AB*, so $M_A = |AD|$ or in other words the length of the segments *AB* and are in proportion M_A with each golden number.

$$\frac{\frac{M_A}{|AB|}}{\frac{\sqrt[2]{5}-1}{2}} = 1$$

Thus M_A is the Golden Mean quantities $x = |AB|\, y\, |AD|$.

When speaking of proportion between two quantities $\frac{a}{b}$. You can think of many times the quantity *b* "fits" in the amount *a*, ie what is the factor by which to multiply the number *b* to be equal to the amount *a*. However, it is not clear geometric

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interpretation of this ratio. If a third quantity (*z*) is introduced so that $\frac{a}{z}=\frac{z}{b}$ must be *z* is nothing but the geometric mean or mean proportional.

Next we show the different ways to calculate an average of two quantities and relate them to the idea of proportion, which ultimately is a way of comparing two quantities.

The Arithmetic mean (M_A) two quantities can be thought of as the midpoint of the sum of these two amounts, we can say if *x* and *y* are two quantities, the arithmetic mean is the midpoint of the new amount $x+y$. Thus the arithmetic mean must satisfy $2M_A = x+y$ which leads us to the equality $M_A + M_A = x+y$ or equivalently $x - M_A = M_A - y$ or in terms of proportions $\frac{x-M_A}{M_A-y}=1$ this last expression indicates that the difference between each of the two quantities and the arithmetic mean is the same, their ratio is one.

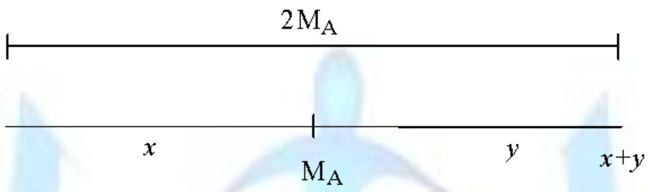

Fig 6

The Geometric Mean M_G between two numbers used to set the rate of growth or decline that occurs between them. It can also be seen as the middle term must go if you take these numbers as terms of a geometric sequence. The geometrical construction of this medium is as follows:

Consider two quantities *x* and *y*, construct the segment *AB* of length $|AB| = x+y$ Find the midpoint of that segment and take it as the center of a circle of radius $r=\frac{x+y}{2}$.

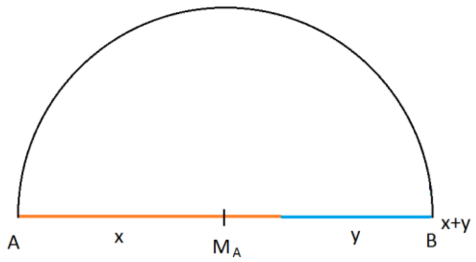

Fig 7

Tracing perpendicular to segment *AB* through point *P* whose distance to point *A* is precisely *x*. Extending this perpendicular until it intersects the circle at point *C* three triangles, triangle *ABC*, the *APC* and the *PBC* are formed.

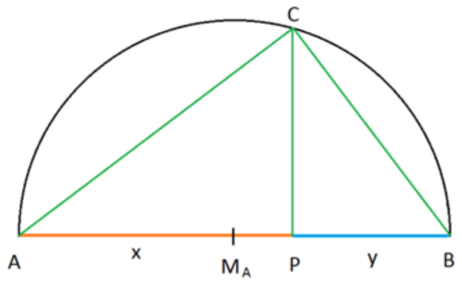

Fig 8

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One of the Theorems of Thales says that the triangle *ABC* is a right triangle, plus the *APC* and *PBC* triangles are similar. So we know that it is satisfied that:

$$\frac{x}{|PC|} = \frac{|PC|}{y} \tag{1}$$

Where *PC* represents the length of the segment of the same name, so that $|PC| = \sqrt[2]{xy}$. This last amount is the geometric mean of the numbers *x* and *y*, this means: $M_G = \sqrt[2]{xy}$. Note that the expression (1) needs to:

$$\frac{xy}{(M_G)^2} = 1$$

This last expression states that the geometric mean of two quantities x and y are such that one is in proportion to the product of them.

Harmonic Media, is defined as follows: If *x* and *y* are two positive numbers the harmonic mean (M_H) is:

$$M_H = \frac{2xy}{x+y}$$

Which can be expressed as:

$$M_H = \frac{\sqrt[2]{xy}\sqrt[2]{xy}}{\frac{x+y}{2}}$$

$$M_H = \frac{(M_G)2}{M_A}$$

That is, the harmonic mean is the rate between the geometric mean squared and the arithmetic mean.

The mean square M_C of two numbers *x* and *y* is determined by:

$$M_c = \sqrt[2]{\frac{x^2+y^2}{2}}$$

or equivalently:

$$M_C = \sqrt[2]{2(M_A)^2 - (M_G)^2}$$

Given two numbers x and y can be set in the following order between the different medias

$$M_H \leq M_G \leq M_A \leq M_C$$

and equality is achieved when *x* = *y.*

Thus, the Pythagoreans decided not to accept the consequences of his reasoning, breaking the permanent bonds that existed between the object and its operational fields. In other words, by not accepting the consequences of operations because it revealed a number property that did not agree with their preconceptions, the Pythagoreans were forced to modify their notion of number, thus excluding the numerical representation of the magnitudes continuous. This led to a separation between arithmetic and geometry, between the domain of the discrete and the continuous domain. The implications of this decision were felt for a long period in the history of mathematics ended in the sixteenth century.

THE PERIOD FROM THE SIXTEENTH TO THE EIGHTEENTH

With the destruction of the Museum of Alexandria, mathematics in general fell into a great abyss, whose production was low and poor for about a millennium. The works of Euclid, Apollonius and Archimedes known only through translations, could be read in direct sources, and a new curiosity was aroused by the geometry, but progress was slow at first, but after the age stage of assimilation, geometrical ideas acquired the abstract and general.

Throughout the sixteenth century and a good part of the eighteenth century the attention of mathematicians headed especially algebra and calculus that was discovered by Newton and Leibniz. So is for Renaissance geometers who cared to give the science of cultivating lacking generality, focusing on the problems of representation of space.

The rendering issues that arose during the Renaissance would become an important opening for the study of transformations. During this historical moment of the transformation function as a useful tool in solving practical problems, especially in painting and architecture are highlighted.

Albrecht Durer (1471-1528), perhaps the most brilliant of the German Renaissance, formed in her Underweysung der Messung (Click for measurement, 1525) the first geometric sketches in which an object was represented by its two projections: horizontal and vertical, and where the similarity explicitly played a key role.

For the perspective of a square *ABCD* located in the horizontal plane, we will continue with the vertices *B, C* and *D* with the same process as *A*. Then *A'B'C'D'* perspective would (or central projection) of the square *ABCD.*

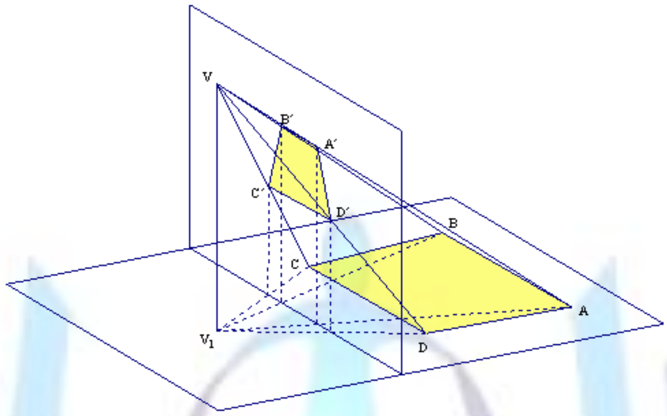

Fig 9. Window projective Durer

Girard Desargues (1591-1662) was one of the first to attempt the task of providing a solid basis to the rules used by Renaissance painters. Thus opened a completely new pathway to deal with the problems of architectural design from purely geometrical procedures. Despite using the resources of descriptive geometry, its nomenclature was often complicated, and found the opportunity or the need to make their brilliant ideas propagate properly.

One of the first discoveries in projective geometry was the famous theorem of Desargues triangles: If two triangles *ABC* and *A'B'C' are* perspective from a point *O* (ie if *AA'*, *BB* 'and *CC'* concur *O*), then the intersections of corresponding sides belonging to the same line (ie $P = AB \cap A'B'$ $Q = BC \cap B'C'$ $R = CA \cap C'A'$ are collinear).

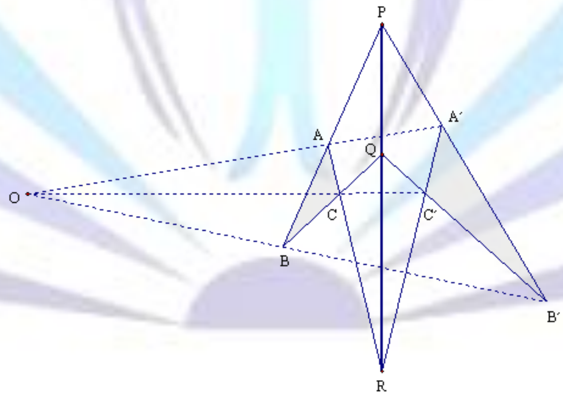

Fig 10

Sean triangles *ABC* and *A' B' C'* perspective, with *O* as center of perspective, and let *AB* and *A' B'* intersect at *P*. *BC* and *B'C'* in *Q* and *CA* and *C'A'* to *R.* Applying Menelaus Theorem to the triangle *ABO* with as transversal *B'A'P* obtain:

$$\frac{AP}{PB}.\frac{BB'}{B'O}.\frac{OA'}{A'A} = -1$$

Similarly, the triangle cross BCO with B'C'Q as it follows that

$$\frac{BQ}{QC}.\frac{CC'}{C'O}.\frac{OB'}{B'B} = -1$$

And the triangle CAO with A'C'R as transverse line, we have that

$$\frac{CR}{RA}\cdot\frac{AA'}{A'O}\cdot\frac{OC'}{C'C}=-1$$

The product of these three equations gives us

$$\frac{AP}{PB}\cdot\frac{BQ}{QC}\cdot\frac{CR}{RA}=-1$$

Demonstrating that *P*, *Q* and *R* are collinear.

The importance of the Desargues theorem in this analysis lies in the properties of shapes that can be classified as metrics and / or projective, setting those properties necessarily related, either directly or implicitly. An example of the former are: equal line segments, the similarity of triangles, of the second, concurrency of lines and collinearity of points.

We can consider Desargues as the initiator of Projective Geometry, their results were written in his book that had the misfortune of not finding the circumstances and the timing for dissemination and development. To this was joins the growing interest towards other branches of the newly created, such as mathematics, Differential and Integral Calculus, which led to the ideas contributed by Desargues fall into oblivion.

NINETEENTH AND TWENTIETH CENTURIES

In the nineteenth century are considered dilation and likeness as mathematical objects, due in large part to the development experienced by the geometry from the date of publication of the descriptive geometry of Monge (1820) and the Erlangen Programme of Felix Klein (1872).

First, in the Normal School of Paris and then at the Polytechnic Garpard Monge (1746-1818) took an active part. In his Treatise on Projective Geometry, Monge sustained early example of the fruitfulness of the transformation of spatial figures in planes, which can prove many propositions of plane geometry through the consideration of figures that are both projections, as well as most of the theorems of transverse and almost all the properties of conics.

Monge The approach taken to determine the position of the center of gravity of a tetrahedron is considered the plane parallel to two opposite edges (*AD* and *BD*, in Fig 10), and decomposing the tetrahedral sheet of infinitesimal thickness.

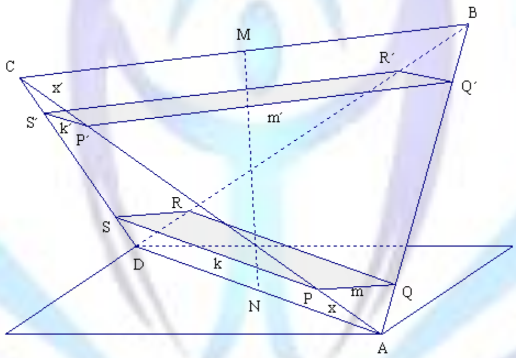

Fig 11

If we cut by a plane parallel to and intersecting the tetrahedron in the quadrilateral *PQRS*, we can state the following:

a) The quadrilateral *PQRS* is a parallelogram, as being the cutting plane parallel to the sides *AD* and *BC*, the *PQ* and *RS* sides must be parallel to *BC*, and the other two: *QR* and *PS* will be part of *AD.*

b) A parallelogram *PQRS* all with *P* at a distance *x* from *A*, corresponds another *P'Q'R'S'* at a distance of *x* of *C*, we will show that they are equivalent. The area of the parallelogram whose adjacent sides are *m* and *k,* and determine an angle α is $A = m.k.sena$. For the second parallelogram area is: $A' = m.k.sena$, since the angle is the same as having the respectively parallel sides *AD* and *BC*. Now we only have to prove that $m.k = m'.k'$.

Is α the length of side *AC.*

The *APQ* and *AP'Q'* triangles are similar, therefore $\frac{m'}{m}=\frac{a-x}{x}$ and similar triangles *CPS* and *CP´S´,* we have: $\frac{k}{k'}=\frac{a-x}{x}$ From which we deduce that $\frac{m}{m'}=\frac{k}{k'}$ where is obtained $m.k = m'.k'$, that is, the two parallelograms are equivalent.

In this sense, the similarity played an important role in this period, while useful tool for solving problems, but was not given recognition as a mathematical object, preserving the static sense. This sense of likeness as a mathematical object was an obstacle that remained for centuries in hibernation. This process culminated in the nineteenth century with Hilbert, with the formulation of the Euclidean axioms and enhancement of the deductive system.

The mathematical developments in the first half of the nineteenth century were influential until the early twentieth. They dominated the field of geometry until such time that the ideas of Sophos Lie emerged and F. Klein.

Klein (in his Erlangen Program in 1872) achieved the synthesis of geometries based on the notion of transformation group, which allows you to enter precise distinctions between different types of geometries. The main group of transformations of the space consists of the set of all transformations that leave invariant geometric properties of figures. Various groups of transformations describing different geometries, enabling the authorities to study the integrated from the point of view of invariant properties in the transformations of each group.

From this viewpoint, the similarity concept as a mathematical object is conceived transformation characterized in that a treatment that seeks the transformation resulting from two or more transformations.

IDENTIFICATION OF EPISTEMOLOGICAL OBSTACLES

The study of problem situations treated in different historical periods, the invariants of those who have taken collective consciousness and different symbolic representations used have allowed us to identify the following epistemological obstacles are described below:

Philosophical area:

a) The Pythagorean School.

The Pythagorean School worked on the vision that saw the world organized on the basis of whole numbers and their reasons, they erected a formidable obstacle. They explored the operational fields of their number concept and brought them to the very limits, only to find there an unexpected result: the incommensurability of two segments. This dismantled his conception of mathematics and, as the two were inseparable, their conception of the world. The expression of nature in mathematical terms-like proportions and reasons-was a key idea in the philosophy developed by the Pythagoreans.

b) The Kantian philosophy.

The epistemology of Kant (1724-1804) is a brilliant synthesis of rationalism and empiricism. Kant asserts that sensory impressions pass through the sieve of a conceptual apparatus (he imagines constituted by forms of sensibility ,) we already have and that gives those impressions , as the container the liquid is poured into it , ways that are comprehensible to our understanding. By emphasizing the power of the mind to organize a world that never fully know , he was paving the way for new structures to the contrary that they were so firmly held in his time. This philosophical view had an impact on contemporary mathematicians creating stagnation in the concept at hand. The concept of similarity as a fundamental concept, used by Wallis in 1663, reappeared in the early nineteenth century with the work of Carnot and say where Laplace's Theory of Parallel has a close relationship with the notion of similarity, whose degree of evidence corresponds approximately equal and that once supported this notion facilitates addressing the Theory of Parallel with rigor.

Methodological area:

a) Separation studio Obstacle number of magnitudes.

This separation of the study of the number of variables, is an obstacle to the notion of similarity. This is evident in Euclid's Elements (300 BC.), for arithmetic and treatment of magnitudes are working independently: Book VII refers to the proportional numbers, and the book to V magnitudes proportional.

Currently associate naturally to any number of numerical magnitude a measure, but in Greek thought magnitudes and numbers were very different objects. The numbers were considered essentially as discrete, while the magnitudes were continuous.

b) Obstacle algebraization Geometry.

The similarity is considered as a simple translation of the algebraic relationship between the elements of a shape in a specific geometric problem. The relationships can be established by this method, strictly correspond to the internal relations between the elements of a given figure.

c) The deleted recognition as a mathematical.

Usage of similarity as a useful tool for solving graphic problems, preserving the static sense. This sense of likeness as a mathematical object was an obstacle that remained for centuries in hibernation. This process culminated in the current era with Hilbert, who has reformulated valued Euclidean axioms and deductive system.

Logical area:

a) Obstacle of reason and proportion.

Proportions were expressed 'rhetorically' to the relationships established (e.g. Thales Theorem), later moving to expressions like a: b: c: d from the Hellenic Mathematical enduring until the fifteenth century.

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b) Obstacle static conception.

The original idea of similarity was contained between the relations of commensurability between homogeneous magnitudes, static this conception of geometry, it caused a intrafigural view of geometry, the idea of transforming a shape in another was absent, as can be seen in the book VI of Euclid's Elements.

SIMILARITY AS A TEACHING RESOURCE

Regarding the treatment of the subject in High School Education in Mexico, likeness, appears as a method for solving geometric constructions from the synthetic point of view. Some influence of Euclidean geometry, where the theorem Thales generates new concepts is appreciated, but not in all cases referred to it, since the justification for the criteria of similar triangles are based essentially on this theorem, excluding homothetic studying figures.

We have identified at this level the approach to similarity within the intrafigural relationship. The figures abound in the revised texts appear in configurations Thales and separate positions.

In summary, for the study of similarity as an object of education, we take into account the results of epistemological analysis, where we identify three different times in the concept "similarity" from them it is possible to identify three approaches that we believe should be present when the similarity is considered as an object of education:

a) Relationship intrafigural. Correspondence between elements of a figure and its corresponding similar in the absence of the idea of transforming a shape in other stands.

b) Geometric Transformation seen as a tool. The geometric transformation is seen as an application of the set of points in the plane itself. Similarity is used as a tool in solving graphic problems.

c) Geometric Transformation as a mathematical object. Characterized in that there is a treatment that seeks the transformation resulting from two or more transformations.

The study aims to show the amount of restrictions placed on teaching the concept of 'similarity' and its movement, which is made at the discretion of the curricular changes that have been implemented in Mexico as part of the Reformation comprehensive High School Education.

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