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Environmentally Sustainable Inventory Model Under Permissible Delay in Payments

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ABSTRACT

Within the economic order quantity (EOQ) framework, the main purpose of this paper is to investigate the supplier optimal replenishment policy of permissible delay in payments. All previously published articles dealing with optimal order quantity with permissible delay in payments assumed that the supplier only offers the retailer fully permissible delay in payments if the retailer ordered a sufficient quantity. Otherwise, permissible delay in payments would not be permitted. However, in this paper, we want to extend this extreme case by developing the mathematical model for determining the cycle time. Under this condition, we model the retailers inventory system as a cost minimization problem to determine the retailers optimal inventory cycle time and optimal order quantity. Finally, numerical examples are given to illustrate all these cases and to draw managerial insights.

Keywords

Inventory; Permissible delay in payments; Cycle time; Transportation cost; Order quantity.

1. INTRODUCTION

We propose an original way to include sustainability criteria into inventory models. Research on sustainability performance has considerably enriched operations management literature in recent years. However, work with quantitative models is still scarce. This paper contributes by revisiting classical inventory models taking environmental sustainability concerns into account. We believe that reducing all aspects of sustainable development to a single objective is not desirable. We thus formulate the classical EOQ model as multiobjective problem. We refer to this model as the sustainable order quantity model under permissible delay in payments.

Mathematical models of inventory typically include the three inventory associated costs of surplus , shortage and costing. These classical models are then analysed as to choose inventory parameters that usually minimize the cost of operating the inventory system being investigated. In particular, the paper identifies a range of inventory problems that are not covered appropriately by traditional inventory analysis. One of these design responsible inventory systems ie) systems that reflects the needs of the environment. However until the cost charged for an activity reflects the true environmental cost of that activity, it is likely that decisions will be made on the basis of erroneous data. In the situation, we are faced with either determining the environmental cost of specific actions or to use environmental costs that are somewhat contrived. This paper discusses these ideas and ways in which inventory models may reassure us with our environmental concerns.

The traditional EOQ model assumed that the supplier must be paid for the items as soon as the items are received. However, in practice a supplier will allow a certain fixed period for settling the amount owed to him for the items supplied. Beyond this period interest is charged. When a supplier allows a fixed time period for settling the account, he is actually given his customer a loan without interest during this period. During the period before the account has to be settled. The customer can sell the items and continue to accumulate revenue and earn interest instead of paying off the overdraft which is necessary if the supplier requires settlement of the account immediately after replenishment. Therefore, it makes economic sense for the customer to delay the settlement of the replenishment account upto the last moment of the permissible delay period allowed by the supplier. Several papers discussing this topic have appeared in the literatures that investigate inventory problems under varying conditions. Some of the prominent papers are discussed below. Goyal [8] established a single item inventory model for determining the economic ordering quantity in the case that the supplier offers the retailer the opportunity to delay his payment within a fixed time period. Chung [5] simplified the search of the optimal solution for the problem explored by Goyal [8]. Aggarwal and Jaggi [2] considered the inventory model with an exponential deterioration rate under the condition of permissible delay in payments. Jamal et al. [13] then further generalized the model to allow for shortages. Hwang and Shinn [12] developed the model for determining the retailers optimal price and lot-size simultaneously when the supplier permits delay in payments for an order of a product whose demand rate is a function of constant price elasticity. Jamal et al. [14] formulated a model where the retailer can pay the wholesaler either at the end of the credit period or later, incurring interest charges on the unpaid balances for the overdue period. They developed a retailers policy for the optimal cycle and payment times for a retailer in a deteriorating-item inventory scenario, in which a wholesaler allows a specified credit period for payment without penalty. Teng [17] assumed that the selling price is not equal to the purchasing price to modify Goyals model [8]. The important finding from Tengs study [17] is that it makes economic sense for a well-established retailer to order small lot sizes and so take more frequently the benefits of the permissible delay in payments. Chung and Huang [6] extended Goyal [8] to consider the case that the units are replenished at a finite rate under permissible delay in payments and developed an efficient solution-finding procedure to determine the retailers optimal ordering policy. Huang [9] extended one-level trade credit into two-level trade credit to develop the retailers replenishment model from the viewpoint of the supply chain. He assumed that not only the supplier offers the retailer trade credit but also the retailer offers the trade credit to his/her customer. This viewpoint reflected more real-life situations in the supply chain model. Khouja [15] showed that for many supply chain configurations, complete synchronization would result in some members of the chain being losers in terms of cost. He used the economic delivery and scheduling problem model and analyzed supply chains dealing with single and multiple components in developing his model. Huang and Chung [II] extended Goyal's model [8] to discuss the replenishment and payment policies to minimize the annual total average cost under cash discount and payment delay from the retailers point of view. They assumed that the supplier could adopt a cash discount policy to attract retailer to pay the full payment of the amount of purchasing at an earlier time as a means to shorten the collection period. Arcelus et al. [3] modeled the retailers profit-maximizing retail promotion strategy, when confronted with a vendors trade promotion offer of credit and/or price discount on the purchase of regular or perishable merchandise. Abad and Jaggi [1] formulated models of seller-buyer relationship, they provided procedures for finding the sellers and buyers best policies under non-cooperative and cooperative relationship respectively. Huang [10] extended Chung and Huang_s model [6], in allowing the retailer adopts different payment policy and finding differences between unit purchase and selling price, and developed an efficient solution-finding procedure to determine the retailers optimal cycle time and optimal order quantity.

All above published papers assumed that the supplier offer the retailer fully permissible delay in payments independent of the order quantity. Recently, Shinn and Hwang [16] determined the retailers optimal price and order size simultaneously under the condition of order-size-dependent delay in payments. They assumed that the length of the credit period is a function of the retailers order size, and also the demand rate is a function of the selling price. Chung and Liao [7] dealt with the problem of determining the economic order quantity for exponentially deteriorating items under permissible delay in payments depending on the ordering quantity and developed an efficient solution-finding procedure to determine the retailers optimal ordering policy. In this regard, Chang [4] extended Chung and Liao [7] by taking into account inflation and finite time horizon. However, all above published papers dealing with economic order quantity in the presence of 912 Y.-F. Huang / European Journal of Operational Research 176 (2007) 911-924 permissible delay in payments assumed that the supplier only offers the retailer fully permissible delay in payments if the retailer orders a sufficient quantity. Otherwise, permissible delay in payments would not be permitted. We know that this policy of the supplier to stimulate the demands from the retailer is very practical. But this is just an extreme case. That is, the retailer would obtain 100% permissible delay in payments if the retailer ordered a large enough quantity. Otherwise, 0% permissible delay in payments would happen. In reality, the supplier can relax this extreme case to offer the retailer partially permissible delay in payments rather than 0% permissible delay in payments when the order quantity is smaller than a predetermined quantity. That is, the retailer must make a partial payment to the supplier when the order is received to enjoy some portion of the trade credit. Then, the retailer must pay off the remaining balances at the end of the permissible delay period. For example, the supplier provides 100% delay payment permitted if the retailer ordered a sufficient quantity, otherwise only a% (0 6 a 6 100) delay payment permitted. From the viewpoint of suppliers marketing policy, the supplier can use the fraction of the permissible delay in payments to agilely control the effects of stimulating the demands from the retailer. This viewpoint is a realistic and novel one in this research field, hence, forms the focus of the present study. Therefore, we ignore the effect of deteriorating item; inflation and finite time horizon similar to most previously published articles.

Under these conditions, We would now develop the mathematical model for determining the cycle time. With this mathematical model if we add the recent costs such as transportation cost which includes fuel cost and road construction cost, then the total relevant cost for the retailer will be increased, we model the retailers inventory system as a cost minimization problem to determine the retailers optimal inventory cycle time and optimal order quantity. Three cases are discussed to describe the optimal replenishment policy for the retailer under the more general framework. Some previously published results of other researchers can be viewed as special cases. Finally, numerical examples are given to illustrate all these cases and to draw managerial insights.

2. NOTATION AND ASSUMPTIONS

In this section, the present study develops a retailer's inventory model under permissible delay in payments. The following notations and assumptions are used throughout this paper.

Notation

D - Demand rate per year.

P - Replenishment rate per year, $P \geq D$

A - Cost of placing one order

ρ - $1-\frac{D}{P} \geq 0$

C - Unit Purchasing Price Per Item

H - Unit Stock holding Cost per Item / Year excluding interest charges

I_e - Interest which can be earned per year

I_K - Interest charges per investment in inventory

M - Permissible delay period

x - Road construction cost per trip

f - Fuel cost

d - Distance travelled

T - Cycle time

Assumptions

1. Demand rate, D is known and constant.
2. Replenishment rate, P is known and constant.
3. Shortages are not allowed.
4. Time period is infinite.
5. $I_K \geq I_e$.
6. During the time the account is not settled generated sales revenue is deposited in an interest bearing account when $T \geq M$ the account is settled at $T = M$ and we start paying for the interest charges on the item in stock.
7. When $T \leq M$, the account is settled at $T = M$ and we do not need to pay any interest charge.

3. MODEL FORMATION

The annual total relevant cost consists of the following elements :

1. Annual ordering cost is not depend upon size and its is calculated for cycle time A/T.
2. Annual Stock Holding Cost depends upon size and the storage space.

$$= \frac{hT(\rho - D)(DT/P)}{2T} = \frac{DTh}{2}\left(1 - \frac{D}{P}\right) = \frac{DTh\rho}{2}$$

3. There are three cases to occur in costs of interest charges for the items kept in stock per year.

<u>Case (i)</u> $M \le PM / D \le T$, shown in fig.1.

Annual interest payable for the goods by the retailer

$$= CI_K \frac{\left[\frac{DT^2\rho}{2} - \frac{(P - D)M^2}{2}\right]}{T} = CI_K \rho \frac{\left[\frac{DT^2}{2} - \frac{PM^2}{2}\right]}{T} \qquad \text{. . . (1)}$$

<u>Case (ii)</u> $M \le T \le PM / D$, shown in fig.2.

$$\text{Annual interest payable} = CI_K \left[\frac{D(T - M)^2}{2}\right] \Big/ T = CI_K \rho \frac{\left[\frac{DT^2}{2} - \frac{PM^2}{2}\right]}{T} \qquad \text{. . . (2)}$$

<u>Case (iii)</u> $T \le M$

In this case, no interest is charged for the items.

4. There are three cases to occur in interest earned per year

<u>Case (i)</u> $M \le PM / D \le T$

$$\text{Annual interest earned} = CI_e \left(\frac{DM^2}{2}\right) \Big/ T \qquad \text{. . . (3)}$$

<u>Case (ii)</u> $M \le T \le PM / D$

$$\text{Annual interest earned} = CI_e \left(\frac{DM^2}{2}\right) \Big/ T \qquad \text{. . . (4)}$$

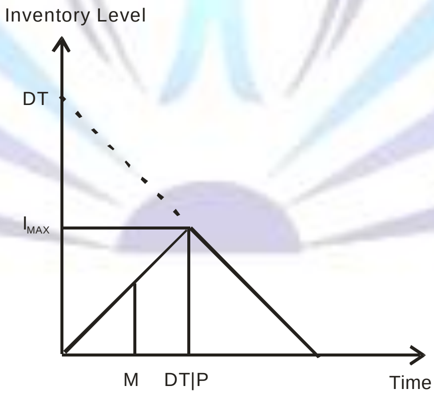

Figure 1. The total accumulation of interest payable when PM/D ≤ T

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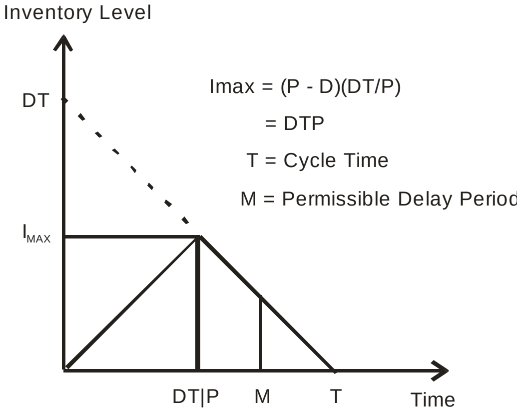

Figure 2. The total accumulation of interest payable when M ≤ T ≤ PM/D

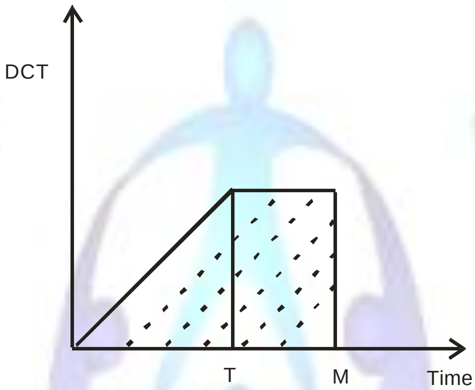

Figure 3. The total accumulation of interest earned when T ≤ M

Case (ii) $T \leq M$ shown in fig.3.

$$\text{Annual interest earned} = CI_e\left[\frac{DT^2}{2} + DT(M - T)\right]\Big/T \qquad \ldots (5)$$

5. Transportation Cost consists of

$$\text{Road Construction Cost / Cycle Time} = \frac{X}{T}$$

$$\text{Fuel Cost / Cycle Time} = f\frac{d}{T}$$

Form the above arguments, the total relevant cost for the retailer can be expressed as

TVC(T) = Ordering Cost + Stock Holding Cost + Interest Payable

– Interest Earned + Transportation Cost

We show that the annual total relevant cost, TVC(T) is given by

$$TVC(T) = \begin{cases} TVC_1 & \text{if } T \geq \frac{PM}{D} \\ TVC_2 & \text{if } M \leq T \leq \frac{PM}{D} \\ TVC_3 & \text{if } 0 \leq T \leq M \end{cases} \qquad \ldots (6)$$

where

$$TVC_1 = \frac{A}{T} + \frac{DTh\rho}{2} + CI_K\rho\left(\frac{DT^2}{2} - \frac{PM^2}{2}\right)\Big/T - CI_e\left(\frac{DM^2}{2}\right)\Big/T + \frac{x}{T} + \frac{fd}{T} \qquad \ldots (7)$$

$$TVC_2 = \frac{A}{T} + \frac{DTh\rho}{2} + CI_K\left(\frac{D(T-M)^2}{2}\right)\Big/T - CI_e\left(\frac{DM^2}{2}\right)\Big/T + \frac{x}{T} + \frac{fd}{T} \quad ...(8)$$

$$TVC_3 = \frac{A}{T} + \frac{DTh\rho}{2} + CI_e\left(\frac{DT^2}{2} + DT(M-T)\right)\Big/T + \frac{x}{T} + \frac{fd}{T} \quad ...(9)$$

$TVC_1(PM/D) = TVC_2(PM/D)$ and $TVC_2(M) = TVC_3(M)$, $TVC(T)$ is continuous and well defined.

$$\frac{\partial}{\partial T}(TVC_1) = -\frac{A}{T^2} + \frac{CI_eDM^2}{2T^2} + \frac{CI_K\rho\,.\,\rho M^2}{2T^2} + D\rho\left(\frac{h+CI_K}{2}\right) - \frac{x}{T^2} + \frac{fd}{T^2}$$

Now $\frac{\partial}{\partial T}(TVC_1) = 0$

$$T = \sqrt{\frac{2A - DM^2C(I_K - I_e) - PM^2CI_K + 2x + 2fd}{D\rho(h+CI_K)}}$$

Now again differentiate TVC_1 with respect to T

$$TVC_1'' = \frac{1}{T^3}\left[2A - M^2(CI_KP\rho + DCI_e) + 2x + 2fd\right] > 0$$

The optimum cycle time $T_1^0 = \sqrt{\frac{2A - DM^2C(I_K - I_e) - PM^2CI_K + 2x + 2fd}{D\rho(h+CI_K)}}$

where $\rho = \left(1 - \frac{D}{P}\right) = \sqrt{\frac{2A - DM^2C(I_K - I_e) - PM^2CI_K + 2x + 2fd}{D\rho(h+CI_K)}}$

Differentiating equation (8) with respect to ‘T’ and equating to zero

$$\frac{\partial}{\partial T}(TVC_2) = -\frac{A}{T^2} - \frac{x}{T^2} - \frac{fd}{T^2} + \frac{Dh\rho}{2} + \frac{CI_eDM^2}{2T^2} + \frac{CI_KD}{2T^2} - \frac{CI_KDM^2}{2T^2} = 0$$

$$T = \sqrt{\frac{2A + DM^2C(I_K - I_e) + 2x + 2fd}{D(h\rho + CI_K)}}$$

Again differentiating TVC_2 with respect to T, we get

$$TVC_2'' = \frac{1}{T^3}\left[2A + DM^2C(I_K - I_e) + 2x + 2fd\right] > 0$$

The optimum cycle time T_2^0 is given by

$$T_2^0 = \sqrt{\frac{2A + DM^2C(I_K - I_e) + 2x + 2fd}{D(h\rho + CI_K)}}$$

Now $TVC_3 = \frac{A}{T} + \frac{DTh\rho}{2} + CI_e\left(DM - \frac{DT}{2}\right) + \frac{x}{T} + \frac{fd}{T}$

Differentiating TVC_3 with respect to T and equating to zero we get

$$\frac{\partial}{\partial T}(TVC_3) = -\frac{A}{T^2} - \frac{x}{T^2} - \frac{fd}{T^2} + \frac{Dh\rho}{2} + \frac{CI_eD}{2} = 0$$

$$T^2 = \frac{2A + 2x + 2fd}{D(h\rho + CI_e)}$$

Again differentiating TVC_3 with respect to T, we get

$$TVC_3'' = \frac{1}{T^3}\left[A + x + fd\right] > 0$$

The optimum cycle time T_3^0 is given by

$$T_3^0 = \sqrt{\frac{2A + 2x + 2fd}{D(h\rho + CI_e)}}$$

The optimum cycle time in all the three periods are

$$T_1^0 = \sqrt{\frac{2A - M^2(CI_K P\rho + DCI_e) - PM^2CI_K + 2x + 2fd}{D\rho(h + CI_K)}}$$

where $2A - M^2(CI_K P\rho + DCI_e) - PM^2CI_K + 2x + 2fd > 0$.

$$T_2^0 = \sqrt{\frac{2A - DM^2C(I_K - I_e) + 2x + 2fd}{D(h\rho + CI_K)}}$$

where $2A - DM^2(I_K - I_e) + 2x + 2fd > 0$.

$$T_3^0 = \sqrt{\frac{2(A + x + fd)}{D(h\rho + CI_e)}}$$

where $2(A + x + fd) > 0$.

The Determination of the Optimal Cycle Time T^0

$$T_1^0 = \sqrt{\frac{2A + DM^2C(I_K - I_e) - PM^2CI_K + 2x + 2fd}{D\rho(h + CI_K)}} \quad ...(10)$$

$$T_2^0 = \sqrt{\frac{2A + DM^2C(I_K - I_e) + 2x + 2fd}{D\rho(h + CI_K)}} \quad ...(11)$$

and $$T_3^0 = \sqrt{\frac{2(A + x + fd)}{D\left(h\rho + CI_e\right)}} \quad ...(12)$$

introduced in the previous section. Then $TVC_i^1\left(T_i^0\right) = 0$ for all i = 1, 2, 3. Furthermore we have the following results.

Numerical examples

To illustrate the results, let us apply the proposed method to solve the following numerical examples.

Example 1 :

Let A = Rs.100/order, D = 3000 units/year, P = 3200 units/year, M = 0.1 year, C = Rs.35/unit, I_K = Rs.0.15/year, I_e = Rs.0.12/year, h = Rs.5/unit/year, x = Rs.75/trip, f = Rs.51/litre, d = 50 km. Therefore, T_1^0 = 1.663 year, T_2^0 = 0.57313 year, T_3^0 = 0.6344 year.

Since $\frac{PM}{D} = \frac{3200 \times 0.1}{3000} = 0.1066$ and $M = 0.1 < 0.1066 < T_1^0$, (ie) $M \le \frac{PM}{D} \le T$

∴ The optimal time under permissible delay in payment is 1.663 year. The optimal quantity for the retailer is 4989 units.

Example 2 :

Let A = Rs.120/order, D = 2800 units/year, P = 3000 units/year, M = 0.7 year, C = Rs.10/unit, I_K = Rs.0.2/year, I_e = Rs.0.05/year, h = Rs.4/unit/year, x = Rs.25/trip, f = Rs.35/litre, d = 10 km. Therefore, T_1^0 = 0.327 year , T_2^0 = 0.6971 year T_3^0 = 0.6912 year and M = 0.7 < T_3^0 = 0.6912 year (ie) T ≤ M.

CONCLUSION

The optimal time under permissible delay in payment is 0.6912 year. In example 1, the condition we discussed in the case (i) $M \leq \frac{PM}{D} \leq T$ is satisfied. So the optimal cycle time under permissible delay in payment is considered to be T_1^0. But in example 2, the case (iii) condition $T \leq M$ is satisfied so the optimal cycle time is T_3^0.

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