GOAL PROGRAMMING APPROACH TO CHANCE CONSTRAINED MULTI-OBJECTIVE LINEAR FRACTIONAL PROGRAMMING PROBLEM BASED ON TAYLOR'S SERIES APPROXIMATION

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ABSTRACT
This paper deals with goal programming approach to chance constrained multi-objective linear fractional programming problem based on Taylor's series approximation. We consider the constraints with right hand parameters as the random variables of known mean and variance. The random variables are transformed into standard normal variables with zero mean and unit variance. We convert the chance constraints with known confidence level into equivalent deterministic constraints. The goals of linear fractional objective functions are determined by optimizing it subject to the equivalent deterministic system constraints. Then the fractional objective functions are transformed into equivalent linear functions at the optimal solution point by using first order Taylor polynomial series. In the solution process, we use three minsum goal programming models and identify the most compromise optimal solution by using Euclidean distance function.

General Terms: Multi-objective linear fractional programming, Goal programming.

Keywords: Goal programming, fractional programming, linear fractional programming, multi-objective linear fractional programming problem, Euclidean distance function, Taylor series.

1. INTRODUCTION
In many real world decision making situation, decision makers (DMs) have to optimize the objective functions which are ratio of two functions of decision variables. This type of optimization problem is called fractional programming problem (FPP) [1]. The objective function of FPP may be represented by the ratio of purchasing cost and selling cost, ratio of the productions of two major crops, ratio of death and birth of people of a certain region, ratio of the full time workers and part time workers, ratio of salary and bonus etc. When both the numerator and denominator are linear functions, then it is called linear FPP and if any one of the numerator or denominator is nonlinear, it is then called nonlinear FPP.

Multi-objective linear fractional programming problem (MOLFPP) consists of multiple linear fractional objectives. MOLFPP is solved by using the variable transformation method due to Charnes and Cooper [2] or by adopting the updating objective function method by Bitran and Noveas [3]. Kornbluth and Steuer [4] developed goal programming algorithm for solving MOLFPP. To overcome the computational difficulties for solving MOLFPP, Luhandjula [5] proposed fuzzy approach to MOLFPP. Dutta et al. [6] extended Luhandjula"s approach and solved MOLFPPs by fuzzy programming technique. Sakawa and Kato [7] studied interactive approach for solving MOLFPPs with block angular structure involving fuzzy numbers. Chakraborty and Gupta [8] developed fuzzy set theoretic approach to MOLFPP by transformation of variables. Pal et al. [9] proposed fuzzy goal programming (FGP) procedure for solving MOLFPP. Guzel and Sivri [10] presented Taylor series based solution procedure for MOLFPP. Toksarı [11] studied Taylor series based approach for dealing with MOFLPP in fuzzy environment. Pramanik and Roy [12] studied FGP models for solving MOLFPP. They [13] also developed priority based FGP models for MOLFFP. Recently, Dey and Pramanik [14] studied goal programming (GP) approach for solving linear fractional bi-level programming problem based on Taylor series approximation.

In the decision making situation uncertainties may occur. Usually, uncertainties are characterized by fuzzily and stochastically described events in the decision making context. Dantzig [15] studied stochastic programming (SP) based on Probability theory. There are two main approaches of SP such as chance constrained programming (CCP) due to Charnes and Cooper [16] and two- stage programming due to Dantzig and Mandansky [17]. In CCP, the constraints are transformed into equivalent deterministic constraints by using the known distribution function.

In this paper chance constrained multi-objective linear fractional programming problem (CCMOLFPP) is considered. The objective functions are ratio of two linear functions. The system constraints are characterized by the random variables of known mean and variance. The random variables are transformed into standard normal variables with zero mean and unit variance. We transform the chance constraints with known confidence level into equivalent deterministic one. Then the fractional objective functions are transformed into equivalent linear functions at the optimal solution point by using first order Taylor polynomial series. In the solution process, we use three GP models and identify the most compromise optimal solution by using Euclidean distance function.

Rest of the paper is organized in the following way. Section 2 presents formulation of CCMOLFPP. Section 3 provides construction of deterministic constraints. Section 4 describes the use of first order Taylor series approximation for linearization. Section 5 is devoted to provide GP formulation for CCMOLFPP. Section 6 explains the use of distance function to identify compromise optimal solution. In Section7, illustrative numerical example is solved in order to show the efficiency of the proposed GP approach. Section 8 presents concluding remarks. Finally, Section 9 presents references used in the paper.

2. FORMULATION OF CCMOLFPP

The objective functions are described as the ratio of two linear functions of decision variables. The objective functions can be represented as:

$$Z_k(\bar{x}) = \frac{\bar{c}_k^T \bar{x} + \alpha_k}{\bar{d}_k^T \bar{x} + \beta_k} \begin{pmatrix} \geq \\ \leq \end{pmatrix} g_k, \quad k = 1, 2, \ldots, K \tag{1}$$

$$\bar{x} \in S = \left\{ \bar{x} \in \Re^n : Pr\left(\bar{A}\bar{x} \begin{pmatrix} \leq \\ = \\ \geq \end{pmatrix} \bar{b}\right) \geq 1 - \alpha, \ \ x \geq 0, \ b \in \Re^m \right\} \tag{2}$$

$$\bar{x} \geq \bar{0} \tag{3}$$

$\bar{x}, \bar{c}_k^T, \ \bar{d}_k^T \in \Re^n$ and α_k, β_k are constants, T denote transposition, g_k is the aspiration level and assume that $d_k^T \bar{x} + \beta_k > 0$ for all $\bar{x} \in S$, S is non empty, convex and compact in $\Re^n$.

3. CONSTRUCTION OF DETERMINISTIC CONSTRAINTS

First, consider the chance constraints of the type: $Pr\left(\sum_{j=1}^{n} a_{ij} x_j \leq b_k\right) \geq 1 - \alpha_k \quad k = 1, 2, \ldots, K.$

The constraints are rewritten as:

$$Pr\left(\frac{\sum_{j=1}^{n} a_{kj} x_j - E(b_k)}{\sqrt{var(b_k)}} \leq \frac{b_k - E(b_k)}{\sqrt{var(b_k)}}\right) \geq 1 - \alpha_k \quad k = 1,2,\ldots,K.$$

$$\Rightarrow \alpha_k \geq 1 - Pr\left(\frac{\sum_{j=1}^{n} a_{kj} x_j - E(b_k)}{\sqrt{var(b_k)}} \leq \frac{b_k - E(b_k)}{\sqrt{var(b_k)}}\right)$$

$$\Rightarrow \alpha_k \geq Pr\left(\frac{\sum_{j=1}^{n} a_{kj} x_j - E(b_k)}{\sqrt{var(b_k)}} \succ \frac{b_k - E(b_k)}{\sqrt{var(b_k)}}\right)$$

$$\Rightarrow \Phi^{-1}(\alpha_k) \geq \frac{\sum_{j=1}^{n} a_{kj} x_j - E(b_k)}{\sqrt{var(b_k)}})$$

$$\Rightarrow \Phi^{-1}(\alpha_k)\sqrt{var(b_k)} \geq \sum_{j=1}^{n} a_{kj} x_j - E(b_k)$$

$$\Rightarrow \sum_{j=1}^{n} a_{kj} x_j \leq E(b_k) + \Phi^{-1}(\alpha_k)\sqrt{var(b_k)} \tag{4}$$

where Φ (.) and Φ^{-1}(.) represent respectively the distribution function and inverse of distribution function of standard normal variable.

Now consider the case when $Pr\left(\sum_{j=1}^{n} a_{kj} x_j \geq b_k\right) \geq 1 - \alpha_k$

$k = 1, 2, \ldots, K$.

As before, the constraints are rewritten as

$$Pr\left(\frac{\sum_{j=1}^{n} a_{kj} x_j - E(b_k)}{\sqrt{var(b_k)}} \geq \frac{b_k - E(b_k)}{\sqrt{var(b_k)}}\right) \geq 1 - \alpha_k, \quad k = 1, 2, \ldots, K$$

$$\Phi\left(\frac{\sum_{j=1}^{n} a_{kj} x_j - E(b_k)}{\sqrt{var(b_k)}}\right) \geq 1 - \alpha_k$$

$$1 - \Phi\left(-\frac{\sum_{j=1}^{n} a_{kj} x_j - E(b_k)}{\sqrt{var(b_k)}}\right) \geq 1 - \alpha_k$$

$$\Phi^{-1}(\alpha_k) \geq -\frac{\sum_{j=1}^{n} a_{kj} x_j - E(b_k)}{\sqrt{var(b_k)}})$$

$$\Phi^{-1}(\alpha_k)\sqrt{var(b_k)} \geq -\left(\sum_{j=1}^{n} a_{kj} x_j - E(b_k)\right)$$

$$\sum_{j=1}^{n} a_{kj} x_j \geq E(b_k) - \Phi^{-1}(\alpha_k)\sqrt{var(b_k)} \tag{5}$$

Let us denote the deterministic constraints (3), (4) and (5) as S`

4. USE OF FIRST ORDER TAYLOR'S SERIES APPROXIMATION FOR LINEARIZATION

First, we find out the ideal solution point for the each objective function individually subject to the deterministic constraints. Suppose, $\bar{x}_k^* = \left(x_{k1}^*, x_{k2}^*, \ldots, x_{kn}^*\right)$ be the ideal solution for the k-th objective function. For the linearization, we use Taylor's series of first order and the series is expanded about the ideal solution points of each objective function. The series can be expressed as:

$$Z_k(\bar{x}) \cong Z_k(\bar{x}_k^*) + \sum_{j=1}^{n} (x_j - x_{kj}^*)\left(\frac{\partial}{\partial x_j} Z_k(\bar{x})\right)_{\text{at } \bar{x} = \bar{x}_k^*}$$

$$= \hat{Z}_k(\bar{x}) \quad k = 1, 2, \ldots, K. \tag{6}$$

Where $Z_k(\bar{x}_k^*) = \max_{S`} Z_k(\bar{x}) = g_k$

5. GP MODEL FORMULATION FOR CCMOLFPP

After linearization of $Z_k(\bar{x})$, we set the aspiration level g_k as individual best solution or ideal solution for each the objective goal. Introducing negative and positive deviational variables the achievement function can be written as:

$$\hat{Z}_k(\bar{x}) + d_k^- - d_k^+ = g_k \quad k = 1, 2, \ldots, K \tag{7}$$

where g_k is the aspiration level of the k-th objective goal and $d_k^- \times d_k^+ = 0$.

Since we consider the individual best solution of the objective function, positive deviation is not possible. Therefore, we use only negative deviational variables. Then (7) can be replaced by

$$\hat{Z}_k(\bar{x}) + d_k^- = g_k \quad k = 1, 2, \ldots, K \tag{8}$$

Now, the minsum GP model for CCMOFPP can be formulated as:

<u>Model-I:</u>

$$\text{Min } \xi = \sum_{k=1}^{K} w_k d_k^- \tag{9}$$

subject to

$$0 \le d_k^- \tag{10}$$

and the constraints (3),(4),(5), and (8). Here, w_k is the associated weight for the k- th objective and the decision makers can fit the weight according to the importance of goals in the decision making context.

<u>Model-II</u> (11)

$$\text{Min } \zeta = \sum_{k=1}^{K} d_k^- \tag{12}$$

subject to the constraints (3),(4),(5),(8),(10). (13)

<u>Model-III</u>

$$\text{Min } \lambda \tag{14}$$

subject to the constraints $\lambda \ge d_k^-$ (15)

(3), (4), (5), (8), (10) (16)

6. COMPROMISE SOLUTION BY USING DISTANCE FUNCTION

To compare the solutions obtained from proposed different GP models, Euclidean function [18] can be defined as:

$$L_2(\eta, k) = \left(\eta_k^2 (1-\mu_k)^2\right)^{1/2} \tag{17}$$

$\eta = (\eta_1, \eta_2, ..., \eta_k)$ denotes vector of attribute attention levels. We assume that $\eta_1 + \eta_2 + ... + \eta_k = 1$. If all the attributes are equal, then η_k = 1/K (k = 1, 2, …, K). For maximization problem, μ_k is denoted by μ_k = (the preferred compromise solution)/ (the individual best solution). For minimization problem, μ_k is denoted by μ_k = (the individual best solution)/ (the preferred compromise solution). The solution for which $L_2(\eta, k)$ is minimal, would be considered as the most compromising optimal solution.

7. NUMERICAL EXAMPLE

The following numerical example is considered to illustrate the proposed approach.

Find $\bar{x}$ (x_1, x_2, x_3) so as to (18)

$$\text{maximize } Z_1(\bar{x}) = \frac{3x_1 + 2x_2 + x_3 - 6}{4x_1 + 10x_2 + 7x_3 + 5} \tag{19}$$

$$\text{maximize } Z_2(\bar{x}) = \frac{2x_1 + x_2 + 8x_3}{4x_1 + x_2 + 9x_3} \tag{20}$$

$$\text{maximize } Z_3(\bar{x}) = \frac{x_1 + 5x_2 + 2x_3 + 6}{3x_1 + 12x_2 + x_3 + 2} \tag{21}$$

subject to

$$\Pr(3x_1 - x_2 + x_3 \le b_1) \ge 1 - \alpha_1 \tag{22}$$

$$\Pr(-2x_1 + x_2 + 7x_3 \le b_2) \ge 1 - \alpha_2 \tag{23}$$

$$\Pr(x_1 + 3x_2 + x_3 \ge b_3) \ge 1 - \alpha_3 \tag{24}$$

The mean, variance and the confidence levels are given below:

$$E(b_1) = 10,\ var(b_1) = 4,\ \alpha_1 = 0.01 \tag{25}$$

$$E(b_2) = 15,\ var(b_2) = 9,\ \alpha_2 = 0.02 \tag{26}$$

$$E(b_3) = 25,\ var(b_3) = 16,\ \alpha_3 = 0.03 \tag{27}$$

Using (4), (5) the chance constraints (22), (23) and (24) can be converted into equivalent deterministic constraints as:

$$3x_1 - x_2 + x_3 \le 14.65 \tag{28}$$

$$-2x_1 + x_2 + 7x_3 \le 21.165 \tag{29}$$

$$x_1 + 3x_2 + x_3 \ge 17.46 \tag{30}$$

The individual best solutions for each objective function subject to (3), (28), (29), and (30) are obtained as:

$$Z_1(\bar{x}_1^*) = 0.2967,\ x_1^* = 6.141,\ x_2^* = 3.773,\ x_3^* = 0; \tag{31}$$

$$Z_2(\bar{x}_2^*) = 1,\ x_1^* = 0,\ x_2^* = 5.82,\ x_3^* = 0; \tag{32}$$

$$Z_3(\bar{x}_3^*) = 0.6004,\ x_1^* = 4.5792,\ x_2^* = 2.9921,\ x_3^{**} = 3.9045 \tag{33}$$

Using Taylor's series approximation (6), the objective functions (20), (21) and (22) can be transformed into new linear objective functions.

$$\hat{Z}_1(\bar{x}) = 0.0269x_1 - 0.0144x_2 - 0.016x_3 + 0.1858, \tag{34}$$

$$\hat{Z}_2(\bar{x}) = -0.1718x_1 - 0.1718x_3 + 1, \tag{35}$$

$$\hat{Z}_3(\bar{x}) = -0.0144x_1 - 0.0397x_2 + 0.0252x_3 + 0.6867 \tag{36}$$

Considering the individual best solution i.e. maximum value as aspiration level, we can write achievement functions as:

$$0.0269x_1 - 0.0144x_2 - 0.016x_3 + 0.1858 + d_1^- = 0.2967, \tag{37}$$

$$-0.1718x_1 - 0.1718x_3 + 1 + d_2^- = 1, \tag{38}$$

$$-0.0144x_1 - 0.0397x_2 + 0.0252x_3 + 0.6867 + d_3^- = 0.6004 \tag{39}$$

Now using the GP models (9), (12, and (14) the obtained solutions compared in the Table 1.

Table1: Comparison of optimal solution obtained from proposed GP Models.

GP Model	μ_k	Euclidean Distance L_2
GP Model I	$\mu_1 = .2569$, $\mu_2 = .6046$, $\mu_3 = .9062$	0.8470
GP Model II	$\mu_1 = .3438$, $\mu_2 = 1$, $\mu_3 = .7589$	0.6991
GP Model III	$\mu_1 = .4460$, $\mu_2 = .8356$, $\mu_3 = .7570$	0.6269

Comparing the distance functions, it is clear that GP Model III offers the most compromise optimal solution.

8. CONCLUSION

In this article, GP based chance constrained multi-objective fractional programming problem with random variables is presented. First order Taylor's series approximation is used to convert the fractional objective functions into linear forms. Three models of minsum GP are presented.. Here only negative deviational variables are required to minimize in order to obtain compromise optimal solution. Therefore computational load is less than conventional goal programming model.

For the further research, priority based GP models may be considered. If the objective functions are fuzzily described, then FGP models [19, 20, 21] may be used. The proposed approach can be extended for multi-objective inventory problems with chance constrained constraints. The proposed concept can be extended for chance constrained multi-objective linear plus linear fraction programming problem.

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