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Solution of a Fuzzy Assignment Problem by using Branch-and-Bound Technique with application of linguistic variable.

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Abstract

In this paper Branch and bound technique is applied to assignment problem with fuzzy cost with objective to minimise cost, fuzzy cost is assumed as triangular fuzzy number, Yager's ranking method has been used for ranking the fuzzy numbers, after transforming assignment problem into a crisp one using linguistic variables, assignment problem is solved by branch and bound technique. To deal with fuzzy assignment problem with qualitative data, linguistic variable helps to convert qualitative data into quantitative data.

Key Words

Fuzzy Assignment problem, Yager's ranking method, branch and bound technique.

1.Introduction

Fuzzy set introduced by Zadeh in 1965, to deal with uncertainty of information. The Assignment problem is a special type of Linear Programming Problem in which objective is to assign number of jobs to equal number of persons with minimum cost (or maximum profit), In this paper, we provided a method to solve Fuzzy Assignment Problem (FAP), with fuzzy cost $\hat{C}_{ij}$. Here objective function is considered as fuzzy number and investigate an assignment problem with fuzzy costs $\hat{C}_{ij}$ represented by linguistic variables which are replaced by triangular fuzzy numbers.

2. Preliminaries

Zadeh in 1965 first introduced Fuzzy set as a mathematical way of representing impreciseness or vagueness in everyday life.

2.1 Definition: A fuzzy set $\bar{A}$, defined on the universal set of real numbers R, is said to be a fuzzy number if its membership function has the following characteristics:

1. $\mu_{\bar{A}}$: R [0,1] is continuous.
2. $\mu_{\bar{A}}(x) = 0$ for all $x \in (-\infty,a] \cup [d, \infty)$.
3. $\mu_{\bar{A}}(x)$ is strictly increasing on [a,b] and strictly decreasing on [c,d].
4. $\mu_{\bar{A}}(x) = 1$ for all $x \in [b,c]$, where a<b<c<d.

2.2 Definition: A Fuzzy number $\bar{A} = (a,b,c)$ is a triangular fuzzy number if its membership function is given by

$$\mu_{\bar{A}}(x) = \begin{cases} (x-a_1) / (a_2 - a_1) & \text{if } a_1 \le x \le a_2 \\ (a_3 - x)/(a_3 - a_2) & \text{if } a_2 \le x \le a_3 \\ 0 & \text{otherwise} \end{cases}$$

2.3 Linguistic Variable

A linguistic variable [1] is a variable whose values are linguistic terms. The concept of linguistic variable is applied in dealing with situations which are too complex or too ill-defined to be reasonably described in conventional quantitative expressions. Linguistic variable can be defined in a subset very high, high, medium low, very low; these values can also be represented by fuzzy numbers.

2.4 Yager's ranking method [3]

$Y(a) = 0.5 \int (C\alpha^L, C\alpha^U)d\alpha$, Where $C\alpha^L$ = Lower α-Level cut, $C\alpha^U$ = Upper α- level cut, Integral over [0,1]

Yager's ranking technique satisfies compensation, Linearity and additive property which provides results that are consistent with human intuition. If $Y(s) \le Y(l)$ then $s \le l$.

The proposed method

The assignment problem can be stated in the form of n x n cost matrix $[C_{ij}]$ of real numbers as given in the following table:

Persons	Jobs				
	1	2	3	....	n
1	C_{11}	C_{12}	C_{13}		C_{1n}
2	C_{21}	C_{22}	C_{33}	....	C_{2n}
...	...	...	...	...	...
N	C_{n1}	C_{n2}	C_{n3}		C_{nn}

Mathematically assignment problem can be stated as

Minimise Z = $\sum_{i=1}^{n}\sum_{i=1}^{n} \text{Cij Xij}$

Subject to,

$\sum X_{ij} = 1$, i = 1,2.......n

$\sum X_{ij} = 1$, j = 1,2......n

Where $X_{ij} = 1$, if the ith person is assigned the jth job

= 0 Otherwise. (1)

Is the decision variable denoting the assignment of the person i to job j. C_{ij} is the cost of assigning the jth job to the ith person. The objective is to minimise the total cost of assigning all the jobs to the available persons (one job to one person).

When the cost or time $\hat{C}_{ij}$ are fuzzy numbers, then the total cost becomes a fuzzy number

$\hat{Z} = \sum_{i=1}^{n}\sum_{i=1}^{n} \text{Cij Xij}$,(i,j=12....n) ,Hence it cannot be minimised directly.

Min $\hat{Z}$ = $\sum_{i=1}^{n}\sum_{i=1}^{n} \text{Cij Xij}$,we apply Yager'' ranking method to get minimum objective value $\hat{Z}$ from the formulation

R($\hat{Z}$) = minimise Z = $\sum_{i=1}^{n}\sum_{i=1}^{n}$ R($\hat{C}_{ij}$) X_{ij}

Subject to,

$\sum X_{ij} = 1$, i = 1,2....n

$\sum X_{ij} = 1$ j = 1,2, ...n

Where $X_{ij} = 1$, if the ith person is assigned the jth job.

= 0 Otherwise (2)

Is the decision variable denoting the assignment of the person i to job j. Ĉij is the cost of assigning the jth job to the ith person. The objective is to minimise the total cost of assigning all the jobs to the available persons (one job to one person).

Since R(Ĉij) are crisp values, problem (1) is obviously the crisp assignment problem of the form (1) which can be solved by the branch and bound technique.

Branch & Bound Algorithm:

In Branch and bound technique applied to assignment problem there are m level number in the branching tree, for root node it is zero, Y be the assignment made in the current node of a branching tree. P_Y^m be an assignment at level m of a branching tree, A be a set of assigned cells up to the node P_Y^m from the root node, V_Y be the lower bound of the partial assignment, A up to P_Y^m such that,

$V_Y = \sum_{i,j\epsilon A}^{n} \text{Cij} + \sum_{i\epsilon x}^{n} \text{Cij} \left(\sum_{j\epsilon y}^{n} \ \text{minCij}\right)$

C_{ij} is the cell entry of the cost matrix with respect to the ith row and jth coloumn, X be the set of rows which are not deleted upto the node P_Y^m from the root node in the branching tree, and Y be the set of columns which are not deleted upto the node P_Y^m from the root node in the branching tree.

Branching Rules:

1. At level m, the row marked as m of the assignment problem, will be assigned with the best column of the assignment problem.
2. If there are tie on the lower bound, then the terminal node at the lower most level is to be considered for further branching.

3. Stopping rule: If the minimum lower bound happens to be at any one of the terminal nodes at the (n-1)th level, the optimality is reached. Then the assignments on the path from the root node to that node along with the missing pair of row-column combination will form the optimum solution.

Numerical Example

Let us consider fuzzy assignment problem (2) with rows representing four person A,B,C,D and columns representing the four jobs job1, job2,job3,job4 with assignment cost varying between \$ 5 to \$ 50. The cost matrix [$\hat{C}$ij] is given whose elements are linguistic variables which are replaced by fuzzy numbers. The problem is then solved by Branch and bound method to find the optimal assignment.

$$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array}\begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{pmatrix} \text{Extremely low} & \text{low} & \text{fairly low} & \text{extremely high} \\ \text{low} & \text{very low} & \text{high} & \text{very high} \\ \text{medium} & \text{Extremely high} & \text{verylow} & \text{extremelylow} \\ \text{veryhigh} & \text{low} & \text{fairlylow} & \text{fairlylow} \end{pmatrix} \end{array} \qquad (3)$$

The linguistic variables showing qualitative data is converted into quantitative data using the following tables. As the assignment cost varies between 0 \$ to 50 \$the minimum possible value is taken as 0 and the maximum possible value is taken as 50.

Extremely low	(0,2,5)
Very low	(1,2,4)
Low	(4,8,12)
Fairly low	(15,18,20)
Medium	(23,25,27)
Fairly High	(28,30,32)
High	(33,36,38)
Very High	(37,40,42)
Extremely High	(44,48,50)

The linguistic variables are represented by triangular fuzzy numbers

$$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array}\begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{pmatrix} (0,2,5) & (4,8,12) & (28,30,32) & (44,48,50) \\ (4,8,12) & (1,2,4) & (33,36,38) & (37,40,42) \\ (23,25,27) & (44,48,50) & (1,2,4) & (0,2,5) \\ (37,40,42) & (4,8,12) & (15,18,20) & (15,18,20) \end{pmatrix} \end{array}$$

To calculate Y(0,2,5) by applying Yager's Ranking method.

The membership function of the triangular fuzzy number (0,2,5) is

μ (x) = (x-0)/(2-0), $0 \le x \le 2$

(x-5)/(2-5), $2 \le x \le 5$

The α- cut of the fuzzy number (0,2,5) is $(C\alpha^L, C\alpha^U) = (2\alpha, 5-3\alpha)$ for which

$Y(\hat{C}_{11}) = Y(0,2,5) = 0.5 \int (C\alpha^L, C\alpha^U) d\alpha = 0.5 \int (2\alpha + 5-3\alpha)\, d\alpha = 2.25$, integral over [0,1]

Proceeding similarly, the Yager's indices for the costs $\hat{C}$ij are calculated as:

$Y(\hat{C}_{12}) = 8$, $Y(\hat{C}_{13})=31$, $Y(\hat{C}_{14}) = 47.5$, $Y(\hat{C}_{22}) = 1.75$, $Y(\hat{C}_{23}) = 81.5$, $Y(\hat{C}_{24}) = 79.5$, $Y(\hat{C}_{31}) = 25$, $Y(\hat{C}_{32}) = 47.5$, $Y(\hat{C}_{33}) = 1.75$, $Y(\hat{C}_{34}) = 2.25$, $Y(\hat{C}_{41}) = 79.5$, $Y(\hat{C}_{42}) = 8$, $Y(\hat{C}_{43}) = 35.5$, $Y(\hat{C}_{44}) = 35.5$

We replace these values for their corresponding $\hat{C}_{ij}$ in (3) and solve the resulting assignment problem by branch and bound technique.

$$\begin{pmatrix} 2.25 & 8 & 31 & 47.5 \\ 8 & 1.75 & 35.75 & 39.75 \\ 25 & 47.5 & 1.75 & 2.25 \\ 39.75 & 8 & 17.75 & 17.75 \end{pmatrix}$$

Further Branching: The four different sub problem under the root nodes are shown in Fig 1 lower bound the solution problems shown on its right hand side

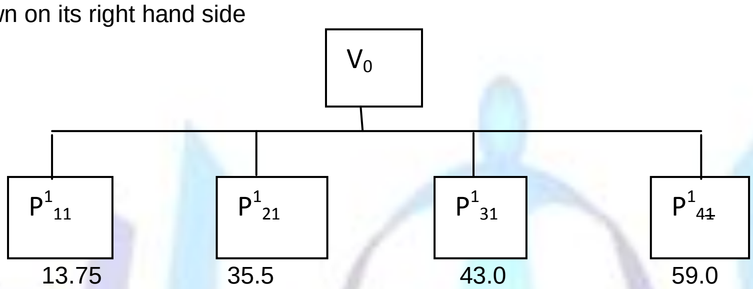

Fig 1

Compute the lower bound for P^1_{11}

$V_Y = \sum_{i,j\epsilon A}^{n} \text{Cij} + \sum_{i\epsilon x}^{n} \quad (\sum_{j\epsilon y}^{n} \ \text{minCij})$

Where Y ={(1,1)} A={(1,1)}, X={2,3,4}, Y={2,3,4}

P^1_{11}=2.25+(1.75+1.75+8)=13.75

P^1_{21}=8+(8+1.75+17.75)=35.5

P^1_{31}=31+(1.75+2.25+8)=43

P^1_{41}=47.5+(1.75+1.75+8)=59

Further Branching: Further branching is done from the terminal node which has the least lower bound at this stage, the nodes P^1_{11}, P^1_{21}, P^1_{31}, P^1_{41} are the terminal nodes. The node P^1_{11} has the least lower bound. Hence, further branching from this node shown in Fig 2

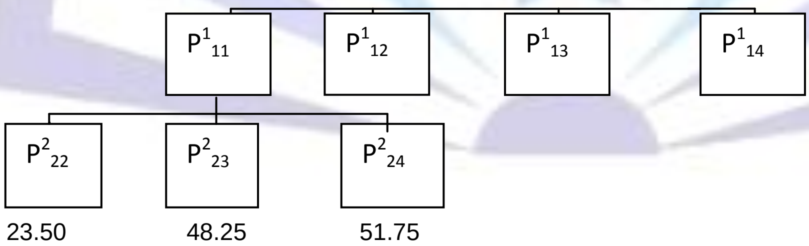

Fig 2

$V(22) = C_{11}+C_{22}+ + \sum_{i\epsilon x}^{n} \quad (\sum_{j\epsilon y}^{n} \ \text{minCij})$

A={(1,1),(2,2), X={3,4} Y={3,4}

P^2_{22}=2.25+1.75+(1.75+17.75)=23.50

P^2_{23}=2.25+35.75+(2.25+8)=48.25

P^2_{24}=2.25+39.75+(1.75+8)=51.75

Further Branching: At this stage the nodes P^1_{11}, P^2_{22}, P^2_{23}, P^2_{24} are the terminal nodes. Among these P^2_{22} has the least lower bound. Hence, further branching from this node is shown in fig 3.

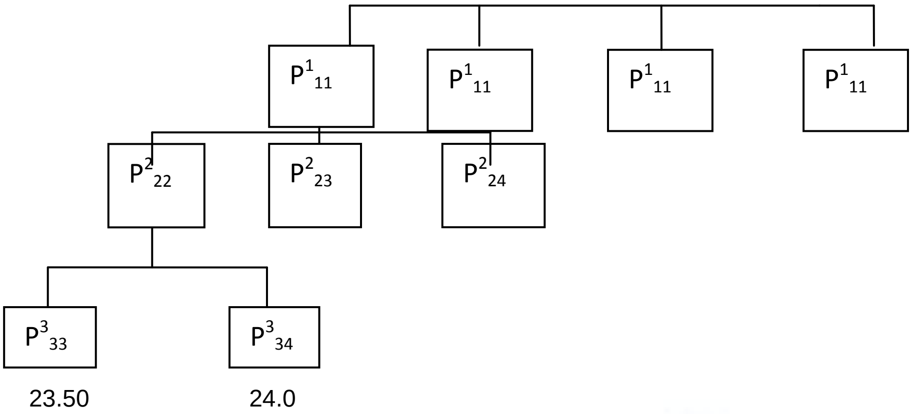

Fig 3

$V(33) = C_{11}+C_{22}+ C_{33}+\sum_{i\epsilon x}^{n} \quad (\sum_{j\epsilon y}^{n} \text{ minCij})$

A={(1,1),(2,2),(3,3), X={4}, Y={4}

P^3_{33}=2.25+1.75+1.75+17.75=23.50

A={(1,1),(2,2),(3,4)},X={4}, Y={3}

P^3_{34}=2.25+1.75+2.25+17.75=24.0

The Optimal assignment schedule is (1,1),(2,2),(3,3),(4,4)

Fuzzy Optimal cost is calculated as (0,2, 5)+(1, 2,4)+(1, 2,4)+(15,18,20)

Also 2.25+1.75+1.75+17.75 = 23.50

Conclusion

In this paper assignment cost is described by fuzzy numbers. Here, the fuzzy assignment problem has been converted into crisp values using linguistic variable and Yager's ranking method. By using branch and bound technique optimal solution is obtained. This method is systematic procedure, easy to apply and can be utilized for all type of assignment problem whether maximize or minimize objective function.

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