Council for Innovative Research
www.cirworld.com

International Journal of Computers & Technology
Volume 4 No. 3, March-April, 2013, ISSN 2277-3061

STUDENTS LEARNING PATHS AS 'DYNAMIC ENCEPHALOGRAPHS' OF THEIR COGNITIVE DEVELOPMENT

Stavroula Patsiomitou
Department of Primary Education, University of Ioannina, Ioannina, Greece
spatsiomitou@sch.gr

ABSTRACT

The current study presents excerpts of my PhD thesis in which I developed a 'dynamic' hypothetical learning path (DHLP) with regard to student's cognitive development. The aim was to investigate if they would raise their van Hiele levels during and after participation in the research process. Students interacted using the interaction techniques of the Geometer's Sketchpad software and the transformations of its dynamic representations. Moreover, the role of linking visual active representations (LVARs) in the software and the role of students' instrumental decoding in the cognitive process have been investigated. Finally, examples of students learning paths, like 'dynamic encephalographs' of their cognitive development, will be presented.

KEYWORDS

'dynamic' hypothetical learning path, dynamic geometry, van Hiele, linking visual active representations, 'dynamic encephalographs'

1. INTRODUCTION

In the sections that follow, I shall present excerpts of my PhD thesis in which I developed a 'dynamic' hypothetical learning path (DHLP) (i.e, a hypothetical learning path through the dynamic geometry software) for the learning of the concept of parallelogram in geometry. I designed [the DHLP] "to engender those mental processes or actions [of students] hypothesized to move [them] through a developmental progression of levels of thinking" (Clements & Sarama, 2004, p.83). In my study I have used the theory of van Hiele (1986) both in the design of the activities in the DGS environment in the light of "the path by which learning might proceed" (Simon, 1995, p.135) and for describing of student's behaviour. The difficulties which arise when a student studies geometry begin with the perceptual competence of a student to 'see' a figure's properties and depends on his/her development of *cognitive structures* and ability to think *abstractly*. Battista (2007) "has elaborated the original van Hiele levels to carefully trace students' progress in moving from informal intuitive conceptualizations of 2D geometric shapes to the formal property-based conceptual system" (p.851) and separated each level in sublevels. For example, Level 2 is separated into sublevels 2.1, 2.2, and 2.3. Level 3 is separated into sublevels 3.1, 3.2, 3.3, and 3.4, each of which has concrete characteristics. The solution of a problem in a DGS environment depends on the preexisting conceptual knowledge of students about figure and their procedural knowledge of the tools and theorems which might be used, moreover the tools' efficiencies. Furthermore, conceptual knowledge of students emanates in response to *instrumental genesis* (e.g., Rabardel, 1995) through the tool use of the software and the development of argumentation as a discursive process, supported by the visualization provided by the dynamic diagram. Consequently, the mathematical language of students and the mathematical meanings or 'dynamic' meanings (Patsiomitou, 2011) evoke as 'dynamic reinvention' through interaction with semi-preconstructed diagrams. By 'semi-preconstructed diagrams', I mean precontsructed diagrams that can be modified during student's *dynamic reinvention* of knowledge (Patsiomitou, 2012a, p. 56), meaning the kind of knowledge the students could reinvent by interacting with the artefacts made in the Geometer's Sketchpad (Jackiw, 1991) DGS environment. In previous studies, I have supported the effect that a concrete type of semi-preconstructed dynamic diagram, the *Linking Visual Active Representations* (see for example Patsiomitou, 2008, 2010) have on the student's gradual competence towards rigorous proof construction, during a problem solving process. Recently, I have extended the conceptual frame of the Linking Visual Active Representations in order to include what emanated from the research process through out in depth data analysis (Patsiomitou, 2012a, b). In the current study I shall present the role of the linking visual active representations (LVARs) of the software to their cognitive process and examples of students learning paths like 'dynamic cardiographs' of their cognitive development will be presented.

1.1 Operational definitions of the theoretical frame

Guitierrez and Jaime (1998) describe student's processes of constructing definitions and justification at every van Hiele level as they develop geometrical thought. This evolution of students' formulation of definitions, justification, and reasoning was adopted by this study as the characteristic that would indicate their movement through several van Hiele levels. For definitions, Govender and De Villiers' (2003) clarification was adopted and extended for purposes of my study. For proof schemes, Balachef's (1998) proof schemes were used. The several kinds of definitions and proof schemes used will be defined. With regard to Govender and De Villiers' (2003, p.47) definitions, the following will be used: (1) *Arbitrary definition*: a different, alternative but correct definition for the same concept. (2) *Sufficient definition*: It contains enough information […] and only those elements of the set we want to define. (3) *Incomplete definitions*: It contains insufficient and incorrect properties (4) *Economical definitions*: It has only necessary and sufficient properties. *Arbitrary and economical definition* is a synthesis of the previous definitions that I have used for purposes of my study. In addition, *dynamic perceptual definition* (Patsiomitou, 2012b) is the term for the process by which the student informally 'defines' a geometrical object by using the tools of the software. With regard to proof schemes reported by Balachef (1998) the following will be used: (1) *Generic example*: the justification is based on operations or transformations on an example which is selected as a characteristic representative of a class. (2) *Thought experiment*: the actions are externalized and dissociated from the specific examples considered. Furthermore, two terms played major role in the analysizing research data (Patsiomitou, 2011, p. 362): (1) *Theoretical dragging* of a drawing into a figure on screen, meaning the student intentionally and successfully transforms a drawing to give it additional properties; and (2) *Instrumental decoding,* meaning competently transforming his/her mental images to actions by using the software interface. Also the meaning of *LVARepresentations* (Patsiomitou, 2012a, p. 76): the successive building steps in a dynamic representation of a problem, or the steps that are repeated in different problems or steps reversing a procedure in the same phase or between different phases of a "dynamic" hypothetical learning path. And finally the meaning of *Reflective Visual Reaction* (Patsiomitou, 2010): the reaction of the student based on reflective thinking due to the interaction with the LVARs of the software, facilitating the construction of meanings and the problem's solution. For the representation of student's argumentation I used a pseudoToulmin's model (Patsiomitou, 2012a, p.57) --based on Toulmin's model (1958)-- in which: (1) the data could be an element or an object of the dynamic diagram, and (2) a warrant could be a tool or a command that guarantees the result which is the claim (or the resulted formulation).

Fig.1. An example of a reduced pseudo-Toulmin's model

In the Figure 1, a segment AB is the data (D), the rotation tool is the warrant (W), and the resulting segment is the claim (C). This means that the rotation tool guarantees the congruency and perpendicularity of the transformed segment on screen. So, students perceive the properties of the rotated segment and during *instrumental gen*esis process they are able to construct the meaning.

2. RESEARCH DESIGN

The *qualitative study* (Merriam, 1998) with a *quasi-experimental design* (Campbell & Stanley, 1963) was conducted in a public high school class in Athens. For the research process twenty eight students volunteers were divided into 'experimental' and 'control' teams, of 14 students each. Students were ages 15 and 16, equal numbers of boys and girls, and all in levels 1 and 2. The students first had been evaluated by their responses to the 20 questions of the 25 multiple-choice questions van Hiele test of Usiskin (1982). The aim of the study was to investigate if the students who had followed the DHLP could develop their thinking and to compare their development with the development of the control group which, had not followed the DHLP. I initially analyzed research data, based on categories described in the theoretical framework of the thesis. The resulting coded text led me to observe and compare of common characteristics that appear in every phase of the DHLP, for students who pretested at the same van Hiele level. So, I isolated every students' story from the entirety of the coded text and analyzed again with regard to the effect of interaction techniques with the software on (Patsiomitou, 2012b): (1) development of the student's competency to translate representations of the same entity and development of their structural analysis of dynamic representations; (2) development of construction and formulation of meanings and the transformation of meanings; and (3) their synthesis of the development of different kinds of reasoning, argumentation (i.e. transformative, inductive, abductive or deductive) and justification (i.e. proof schemes). Moreover, the definitions of *theoretical and experimental dragging* and the meaning of *instrumental decoding* have played a major role in data analysis. In the present paper I shall focus on a pair's movement through the van Hiele levels during their interaction with the activities of the DHLP. M_1 is a male pupil (van Hiele level: 1 at the pre-test) and M_2 is a female pupil (van Hiele level: 2 at the pre-test). I shall describe their movement through van Hiele levels and I shall use an Excel matrix in which we can visualize their evolution.

Research question*: Does student's instrumental decoding of the tools use in a DHLP ('dynamic' hypothetical learning path) supported by LVARs (Linking Visual Active Representations) affect students' cognitive development?*

3. VISUALIZATION OF STUDENTS' COGNITIVE DEVELOPMENT

Visualization of student's cognitive development during the research has been made by pointing out the main snapshots of their development in an Excel matrix. Concretely, I conceived and applied the following process: The first column of the matrix mentioned above contains the tools and synthesis of tools that helped students to formulate an expression or a characteristic that could be an indication of their van Hiele level. Tools were categorized and every tool defined by a distinct code (Patsiomitou, 2012b): T1 for the point used by experimental dragging, T2 for the point used by theoretical dragging, T3 for the reflection tool, T8 for the circle tool, T9 for the rotation tool, T11 for the parametric tool, T12 for the custom tool "symmetry" (Patsiomitou, 2012a, p. 68), T15 for the custom tool used with 'economy or catachresis', T16 for the hide/show action button tool, T17 for the trace tool, and T18 for the annotation tool. Furthermore, the characteristics of the van Hiele levels that appeared during analysis of students' dialogues led me to create an adaptation to Battista's (2007) categorization. Concretely, the first row of the Excel matrix contains characteristics of the van Hiele levels, each with a distinct code (Patsiomitou, 2012b). For example, characteristics of level 1, 2.1 […] 3.4 were coded as follows: I0 for *cognitive conflicts* and I1 for *informal descriptions*, etc. described at the Table 1 below.

Table 1. An adaptation to Battista's (2007) categorization on van Hiele levels

level 1	I0 for cognitive conflicts and I1 for incorrect and informal descriptions.
level 2.1	I2 for *dynamic perceptual definition* and I3 for the synthesis of formal and informal descriptions of students.
level 2.2	I4 for *incomplete definitions* and *incomplete* reasoning and I5 for inductive argumentation/concepts-in-action or theorems-in-action.
level 2.3	I6 for formal description and *non-economical definitions* and I7 for connections between meanings.
level 3.1	I8 for *economical definitions* and I9 for logical correlations between meanings.
level 3.2	I10 for structural analysis competence, I11 for abductive-deductive reasoning.
level 3.3	I12 for deductive argumentation, I13 for the *generic example* proof scheme.
level 3.4	I14 for *thought experiment* proof scheme, I15 for the competence of logical hierarchy.

Transformative reasoning, dynamic reinvention, and reflective visual reaction were coded as I16, I17, and I18, respectively. I posed the dialogue counting of the concrete team in which the student participated at the intersection of an intelligible parallel line, starting at the tool and moving to the horizontal axis, and an intelligible perpendicular line, starting from the van Hiele characteristic. The process was accomplished by tracing a crooked line through the counting so that the learning path of the student could be visualized; moreover, it would determine what tools affected to the student's movement at van Hiele levels.

3.1 Students 'dynamic encephalographs' through van Hiele levels

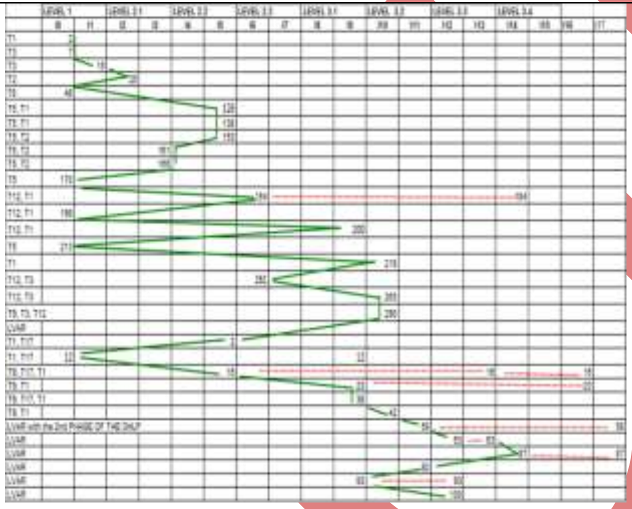

Fig.2: Student's M1 development through van Hiele levels in an Excel screenshot

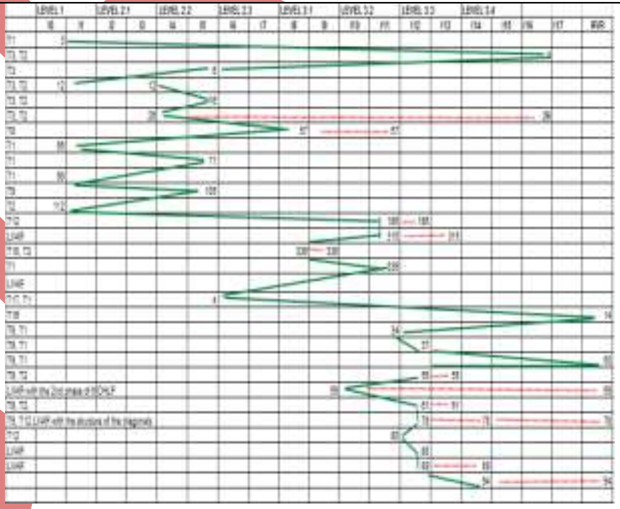

Fig.3: Student's M2 development through van Hiele levels in an Excel screenshot

At the beginning of research, M1 did not have the competency to translate a verbal statement of a problem to an iconic representation or to translate his mental image of the geometric object to a figure. During the research process he faced (1) *cognitive obstacles* with regard to geometrical meanings, as he did not have the competency to connect his construction process with a definition or a geometric theory (2) *instrumental obstacles* due to trying to memorize the use of the software tools. Many times, he faced *cognitive conflicts* [see, for example, points 2, 7, 46, 170, 198, and 213 in Figure 2] when performing experimental dragging and when interacting with the reflection tool and the custom tool 'symmetry'. Through use of the reflection tool, M1 acquired "an increasing ability" in examing the structure of the shapes by analyzing shape components, which is a characteristic of Level 2, according to Battista's (2007, p. 851) categorization of van Hiele levels. For example I dragged the endpoint of a reflected segment AB until it touched the reflection line. M1 recognized "a right triangle and an isosceles triangle", meaning, he recognized the subfigures into which an isosceles triangle is separated by the reflection line, although the isosceles had an unusual orientation on the screen. Subsequently, M1 has developed the competency to perceptually recognize the components by which the figure is analyzed. Also he acquired the competency to translate between representational systems due to the mediation of the software tools. Moreover, the reflection tool contributed to development of his inductive reasoning and theorems-in-action (Vergnaud, 1998). It also was observed that he formulated concepts-in-actions (Vergnaud, 1998) [see, 125, 138 in the diagram below], to develop the competency of logical correlations of figure's properties by using the custom tool and the competency to formulate non-economical definitions during the second phase of the DHLP. For example, he formulated "*In order for point O to be the center of symmetry, it would be this segment congruent with this segment*." This means that, M1 verbally decoded the iconic information, based on his visual perception and on mental transformations of visual data comparison. The central point of the research was the point in the dialogue [200] at which he decoded the verbal representation to a figural one, using a synthesis of formal and informal expressions by using the custom tool ‘symmetry’ mentioned above. This means, that linking the visual representations that the student constructed during the research process became the means to manage the solution of the problems (for example, the construction of a square as a result of reversal of actions using the symmetry features of the figure). In the fourth phase of the DHLP, M1 formulated *economical definitions* in correlation with M2. Also during the fourth phase due to the use of LVARs and the Reflective Visual Reaction (RVR) he had obtained, he developed deductive reasoning, a ‘*generic example’* (Balachef, 1998) as a decoding process in his competence at synthesizing tool use. For example, he formulated “*if we prove that these are parallel lines, then the quadrilateral is a parallelogram because these are equal, so the diagonals will be intersected, so the diagonals will be dichotomized”.* In particular, the rotation tool and theoretical dragging of the figure’s points in the fourth phase of the DHLP helped him to develop RVR and *dynamically reinvent* the solution to the problem. The other student, M2 faced instrumental obstacles to decoding her mental representation to an iconic one. This student faced *cognitive conflicts* when using the point tool. Experimental or theoretical dragging with the tool mediated development of this student’s competency to translate a mental representation to verbal and iconic ones. In the fourth phase of the DHLP she developed competency to mentally connect representations that she had interacted with in previous phases of the DHLP, competency to structurally analyze shapes, and competency to recognize the *dual status* of geometric objects in a problem solving situation. The rotation tool helped M2 to formulate deductive arguments, to logically correlate components of the dynamic representation, and to connect meanings. The points [14, 59, and 80] are points of *dynamic reinvention* of knowledge. For example, M2 formulated “*IPQG is a trapezium because PI and QG are perpendiculars to IG as we concluded from the rotation for* 90^o*. …we must prove that X is the midpoint of any segment that can be. ..These (pointing to PQ, HL) seem to be diagonals but where is the quadrilateral …If we prove that PQ and HL are the diagonals of a parallelogram then the diagonals are dichotomized”.* (Figure 4). RVR of custom tool use during the second phase of DHLP played a major role in developing concrete formulation. M2 also developed the competency to pose subaims and to develop representations conceptually and procedurally linked with or connected to the first and second phases of research. Figure 4 shows the role of the tools in developing M1’s and M2’s complex deductive argumentation during research in the fourth phase of the LVARs.

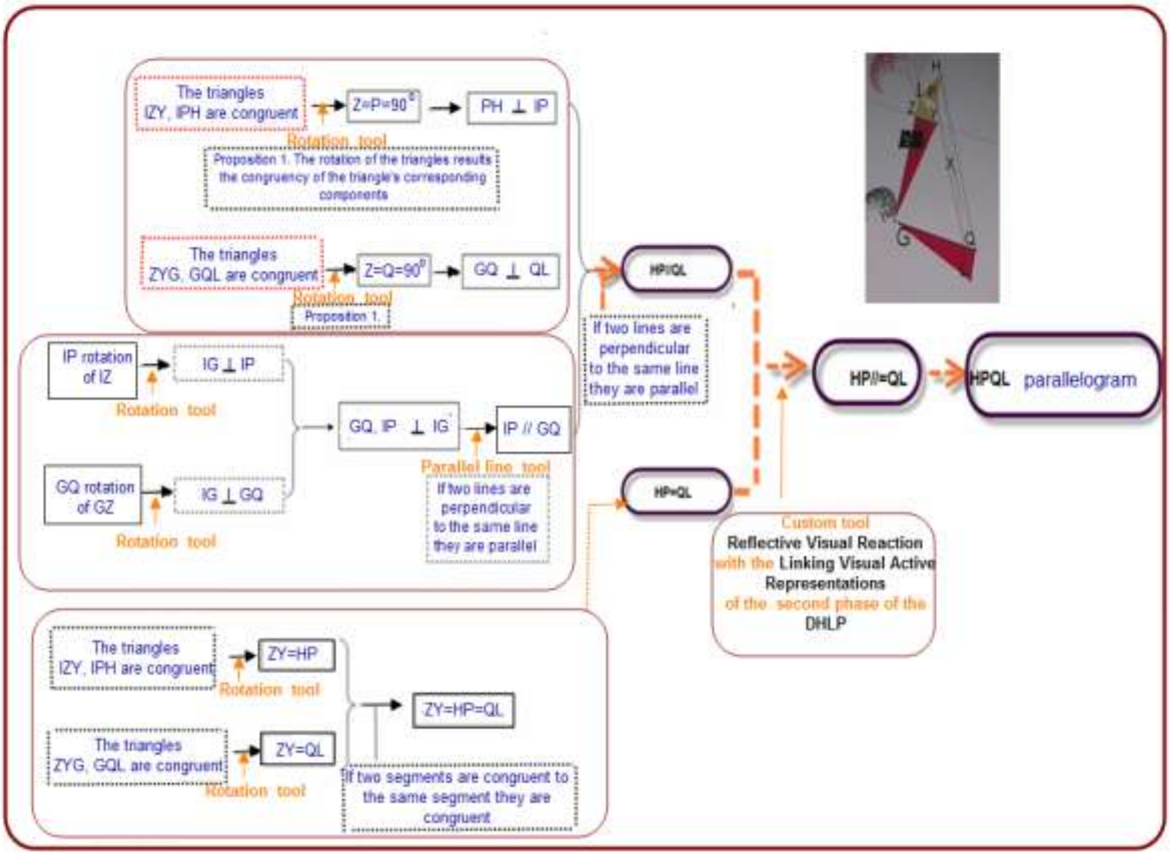

Fig. 4. Students’ deductive argumentation during the fourth phase of the DHLP

4. CONCLUSIONS

The Excel matrix results for students M1 and M2 presents the students’ evolution and movement through van Hiele levels. It shows that student M1, M2 has developed his level during the fourth phase of DHLP, and indicates achievement of Level 3.2-3.3. Student M2 also has developed her level during the fourth phase of DHLP and achieved Level 3.4. Consequently, results guarantee that a student’s thought can be developed in the organizing framework of DHLP by LVARs in which the student participated. Finally, “it is important for mathematics education researchers to heed the work of other researchers in other fields such as cognitive science and neuroscience [in order to] make true progress in this vital area of research" (Battista, 2007, p. 859).

REFERENCES

[1] Ball, D. L. (1990). Prospective elementary and secondary teachers' understanding of division. *Journal for Research in Mathematics Education*, *21*(2), 132-144.

[2] Balacheff, N. (1998). Construction of Meaning and Teacher Control of Learning. In T. D & J. D.C. (Eds.), *Information and communication technology in school mathematics* : Chapman & Hall.

[3] Battista, M. T. (2007).The development of geometric and spatial thinking.In Lester, F. (Ed.), *Second Handbook of Research on Mathematics Teaching and Learning* (pp. 843-908). NCTM. Reston, VA: National Council of Teachers of Mathematics.

[4] Clements, D. & Sarama, J. (2004). Learning trajectories in mathematics education. *Mathematical Thinking and Learning, 6(2),* 81-89.

[5] Campbell, D. & Stanley, J. 1963, 'Experimental and quasi-experimental designs for research on teaching', in Gage, N., (ed) *Handbook of Research on Teaching*, Chicago, Rand McNally

[6] Govender, R. & De Villiers, M. "A dynamic approach to quadrilateral definitions." Pythagoras 58 (2004): 34-45.

[7] Gutierrez, A.& Jaime, A. (1998). On the assessment of the Van Hiele levels of reasoning, *Focus on Learning Problems in Mathematics 20* (2/3), 27-46. www.uv.es/Angel.Gutierrez

[8] Jackiw, N. (1991). *The Geometer's Sketchpad* [Computer Software].Berkeley, CA: Key Curriculum Press

[9] Merriam, S. B. (1998). *Qualitative research and case study applications in education*. San Francisco: Jossey-Bass

[10] Patsiomitou, S. (2008). The development of students' geometrical thinking through transformational processes and interaction techniques in a dynamic geometry environment. *Issues in Informing Science and Information Technology Journal*, 5, 353-393. Available on line http://iisit.org/IssuesVol5.htm

[11] Patsiomitou, S. (2010). Building LVAR (Linking Visual Active Representations) modes in a DGS environment. *Electronic Journal of Mathematics and Technology,* 4(1), 1-25.

[12] Patsiomitou, S. (2011). Theoretical dragging: A non-linguistic warrant leading to dynamic propositions. *Proceedings of the 35th Conference of the International Group for the Psychology of Mathematics Education*, Vol. 1, pp. 361-368. Ankara, Turkey: PME

[13] Patsiomitou, S. (2012a). A Linking Visual Representation DHLP for student's cognitive development. *Global Journal Of Computer Science and Technology*. 12 (6), 53-82.

[14] Patsiomitou, S. (2012b).The development of students' geometrical thinking trough transformational processes and interaction techniques in a dynamic geometry environment. Linking Visual Active Representations. PhD thesis. *Department of Primary Education of Ioannina. University of Ioannina,* December 2012.

[15] Rabardel, P. (1995). Les hommes et les technologies, approche cognitive des instruments contemporains. Paris : Armand Colin.

[16] Simon, M. A. (1995). Reconstructing mathematics pedagogy from a constructivist perspective. *Journal for Research in Mathematics Education, 26* (2), 114-145.

[17] Toulmin, S.E. (1958). *The uses of argument*. Cambridge: Cambridge University Press.

[18] Usiskin, Z. (1982). *Van Hiele levels and achievement in secondary school geometry*. Chicago: University of Chicago. (ERIC Document Reproduction Service,No.ED 220 288).

[19] Vergnaud, G. (1998). Comprehensive theory of representation for mathematics education. *Journal of Mathematics Behavior*, 17(2), 167-181.

Author' biography with Photo

Dr. Stavroula Patsiomitou is a mathematician working in the state secondary education system. She was awarded her PhD in the Didactics of Mathematics at the University of Ioannina, Department of Primary Education, in 2012. The topic of her thesis was "The development of students' geometrical thinking through transformational processes and interaction techniques in a dynamic geometry environment: Linking Visual Active Representations." Prior to this, in 2005, she earned her Master's degree in Education, specifically, in the Didactics and Methodology of Mathematics, in the inter-university program of the University of Athens (NKUA) and the University of Cyprus. She has written an algebra textbook in Greek for 16- to 18-year-olds and a dynamic geometry textbook (two volumes) in Greek (titled *Learning mathematics with the Geometer's Sketchpad v4*) which was approved by the Pedagogical Institute and has been sent to experimental high schools in Greece. It consists of 15 stand-alone chapters which are connected through the basic idea of the development of structures, designed by linking visual active representations. She has also authored or co-authored and presented 45 papers at conferences in Greece and abroad, published 13 articles in refereed journals in both the Greek and English languages, and has written many articles for the journals of the Hellenic Mathematical Society. She was involved in the Greek translation of the software, advising the translation team on the suggestion of its Chief Technology Officer, Nicholas Jackiw. Her name is included on the splash screen "Special Thanks to ..." of the Greek version of the Geometer's Sketchpad v.4 dynamic geometry software. Her research interests centre on computer assisted mathematics learning and teaching in general and dynamic geometry software in particular.