Spline Computation for Solving Magnetohidrodynamics Free Convection Flow

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ABSTRACT

In this paper, we construct numerical algorithms for solving Magnetohidrodynamics(MHD) free convection flow rate which has been discussed in detail. It is observed that, for a nonlinear system of differential equation, the spline model of nine degree is used, effectiveness and accuracy of this new method are presented. Several theorems relating the order of the ninth spline to the saturation of the estimator are proved. Some results of consistency are given and an application MHD system is given.

AMS SUBJECT CLASSIFICATION: 41A25, 65H10, 47E05.

KEYWORD: Splineinterpolation, Convergence analysis, MHD convection flow,System of nonlinear ordinary differential equations.

INTRODUCTION

Consider the system of nonlinear equations

$$\frac{dA}{dt} = f(t, u_1, u_2, u_3, ..., u_n)$$

$$A(t_0) = \eta \tag{1}$$

where $A = [u_1, u_2, u_3, ..., u_n]^T$, $f = [f_1, f_2, f_3, ..., f_n]^T$, $\eta = [\eta_1, \eta_2, \eta_3, ..., \eta_n]$.

Many problems in applied sciences and engineering are modeled as system of differential equations such as spring-mass systems, bending of beams, Magnetohidrodynamics free convection flow (MHD), chemical reactions and so forth can be formulated in terms of differential equations. Since the system of differential equations has wide applications in scientific research, so we consider the convective flow in fluid saturated porous medium which has been the subject of several recent papers. Therefore faster and accurate numerical solutions to this problem is very important, see([1-3], [5], and [12]).

There are several methods that can be used to solve the nonlinear problems numerically. A broad class of analytical solutions methods, such as Runge-Kutta of order six, Taylors series, Hirota's bilinear scheme and Hereman's method as [6-8], [10] and[12], were used to handle these problems. However, some of spline approximation had been proposed by ([2], [4] and [11]]) solved the system of differential equations and some initial value problems.

In the present paper, we discussed the convergence analysis of the ninth spline method for system of differential equations with new constraint and boundary conditions, also we apply this new model for solving magnetohidrodynamics free convection flow and mass transfer over a stretching sheet which has been analyzed numerically including the dufour and sort effects. We will use the function, two, fourth and seventh boundary conditions, to constructed the ninth spline with two initial conditions.

DESCRIPTION OF THE METHOD

We present a ninth spline interpolation for one dimensional and for a given sufficiently smooth function $f(x)$ defined on the interval $I = [a, b]$, and $\Delta_n : a = x_0 < x_1 < x_2 < .... < x_n = b$, denote the uniform partition of I with knots $x_i = a + ih$, where $i = 0, 1, 2, ..., n-1$ and $h = \frac{b-a}{n}$ is the length of each subintervals, and d the ninth spline is denoted by $S_\Delta(x)$ and defined on I as:

$$s_0(x) = y_0 + (x - x_0)y_0' + \frac{(x-x_0)^2}{2}y_0'' + \frac{(x-x_0)^3}{6}y_0''' + \frac{(x-x_0)^4}{24}y_0^{(4)} + \frac{(x-x_0)^5}{120}a_{0,5} +$$

$$(x-x_0)^6 a_{0,6} + \frac{(x-x_0)^7}{5040}y_0^{(7)} + (x-x_0)^8 a_{0,8} + (x-x_0)^9 a_{0,9} \tag{2}$$

on the subinterval $[x_0, x_1]$ where $a_{0,j}$, $j = 5, 6, 7$ are unknowns to be determined.

Let us examine subintervals $[x_i, x_{i+1}]$, i=1,2,..., n-2. By taking into account the interpolating conditions, we can write the expression, for $S_i(x)$ in the following form, see([1-2], [10] and [11]).

$$s_i(x) = y_i + (x - x_0)a_{i,1} + \frac{(x-x_i)^2}{2}y_i'' + (x-x_i)^3 a_{i,3} + \frac{(x-x_i)^4}{24}y_i^{(4)} + (x-x_i)^5 a_{i,5} +$$

$$(x-x_i)^6 a_{i,6} + \frac{(x-x_i)^7}{5040}y_i^{(7)} + (x-x_i)^8 a_{i,8} + (x-x_i)^9 a_{i,9}, \tag{3}$$

where $a_{i,j}$, $i = 1(1)(n-1)$, $j = 1, 3, 5, 6, 8, 9$. are unknown values to be determine.

CONVERGENCE ANALYSIS

In this section, we investigate the convergence analysis of the method for ninth degree spline function which are developed by [2], [4], [10] and [11]. The equation (2) yields the two initial conditions $S'(x_0) = y'(x_0)$ and $S'''(x_0) = y'''(x_0)$, and then we apply the boundary conditions $S^{(r)}(x_i) = y^{(r)}(x_i)$, $r = 0, 2, 4, 7$, for $i = 0, 1, ..., n$ in equation (3), to obtain the following:

$$a_{0,5} = \frac{198}{41h^5}[y_1 - y_0] - \frac{198}{141h^4}y_0' - \frac{239}{410h^2}y_0''' - \frac{1}{410h^3}[91y_1'' + 899y_0''] +$$

$$\frac{h}{4920}[25y_1^{(4)} - 469y_0^{(4)}] - \frac{h^2}{516600}[12y_1^{(7)} - 35y_0^{(7)}],$$

$$a_{0,6} = -\frac{192}{41h^6}[y_1 - y_0] + \frac{192}{41h^5}y_0' + \frac{617}{1230h^3}y_0''' + \frac{1}{3099600h^4}[864360y_1'' + 6393240y_0'']$$

$$-\frac{1}{3099600h}[23100y_1^{(4)} - 195720y_0^{(4)}] + \frac{h}{3099600}[122y_1^{(7)} - 595y_0^{(7)}],$$

$$a_{0,8} = \frac{45}{41h^8}[y_1 - y_0] - \frac{45}{41h^7}y_0' - \frac{9}{82h^5}y_0''' - \frac{1}{826560h^6}[60480y_1''' + 393129y_0''']$$

$$+\frac{1}{36400h^3}[20y_1^{(5)} + 25y_0^{(5)}] - \frac{1}{1834560h}[64y_1^{(7)} + 161y_0^{(7)}],$$

and

$$a_{0,9} = \frac{-10}{41h^9}[y_1 - y_0] + \frac{10}{41h^8}y' + \frac{1}{41h^6}y_0''' + \frac{1}{2479680h^7}[40320y_1''' + 262080y_0''']$$

$$-\frac{1}{2479680h^5}[1680y_1^{(4)} - 6720y_0^{(4)}] + \frac{1}{2479680h^2}[29y_1^{(7)} + 35y_0^{(7)}].$$

Substituting these values of $a_{0,5}$, $a_{0,6}$, $a_{0,8}$ and $a_{0,9}$ in equation (2), we obtain:

$$a_{1,1} = \frac{108}{41h}[y_1 - y_0] - \frac{67}{41}y_0' - \frac{13h^2}{205}y_0''' + \frac{h}{410}[51y_0'' - 181y_0''] -$$

$$\frac{h^3}{4920}[5y_1^{(4)} + 21y_0^{(4)}] + \frac{h^6}{4132800}[11y_1^{(7)} - 15y_0^{(7)}],$$

and

$$6a_{1,3} = \frac{-1080}{41h^3}[y_1 - y_0] + \frac{1080}{41h^2}y_0' + \frac{67}{41}y_0''' + \frac{1}{41h}[154y_1'' + 386y_0''] +$$

$$\frac{h}{246}[23y_1^{(4)} + 31y_0^{(4)}] - \frac{h^4}{103320}[13y_1^{(7)} - 14y_0^{(7)}].$$

We shall find the coefficients of $S_i(x)$ for $i = 1, 2, 3, ..., n-1$. From equation (3) we have:

$$a_{i,5} = \frac{198}{41h^5}[y_{i+1} - y_i] - \frac{198}{41h^4}a_{i,1} - \frac{717}{205h^2}a_{i,3} - \frac{1}{516600h^3}[114660y_{i+1}'' + 113274y_i'']$$

$$+\frac{1}{516600h}[2625y_{i+1}^{(4)} - 49245y_i^{(4)}] - \frac{h^2}{516600}[12y_{i+1}^{(7)} - 35y_i^{(7)}],$$

$$a_{i,6} = \frac{-192}{141h^6}[y_{i+1} - y_i] + \frac{192}{41h^5}a_{i,1} + \frac{617}{205h^3}a_{i,3} + \frac{1}{3099600h^4}[864360y_{i+1}'' + 6393240y_i'']$$

$$-\frac{1}{3099600h^2}[23100y_{i+1}^{(4)} - 195720y_i^{(4)}] + \frac{1}{3099600}[122y_{i+1}^{(7)} - 595y_i^{(7)}],$$

$$a_{i,8} = \frac{45}{41h^8}[y_{i+1} - y_i] - \frac{45}{41h^7}a_{i,1} - \frac{27}{41h^5}a_{i,3} - \frac{1}{826560h^6}[60480y_{i+1}'' + 393120y_i''] +$$

$$\frac{1}{826560h^3}[2520y_{i+1}^{(4)} 10080y_i^{(4)}] - \frac{1}{826560h}[23y_{i+1}^{(7)} + 73y_i^{(7)}],$$

and

$$a_{i,9} = \frac{-10}{41h^9}[y_{i+1} - y_i] + \frac{10}{41h^8}a_{i,1} + \frac{6}{41h^6}a_{i,3} + \frac{1}{2479680h^7}[60480y_{i+1}'' + 262080y_i'']$$

$$-\frac{1}{2479680h^4}[1680y_{i+1}^{(4)} - 6720y_i^{(4)}] + \frac{1}{2479680h^2}[29y_{i+1}^{(7)} + 35y_i^{(7)}].$$

Substituting the values of $a_{i,5}, a_{i,6}, a_{i,8}$ and $a_{i,9}$ in the equation (3), we obtain the following relation, for $i = 1,2,...,n-1$.

$$a_{i+1,1} = \frac{108}{41h}[y_{i+1} - y_i] - \frac{108}{41}a_i - 78\frac{13h^2}{205}a_{i,3} + \frac{h}{410}[51y''_{i+1} - 181y''_i] -$$

$$\frac{h^3}{4920}[5y^{(4)}_{i+1} + 21y^{(4)}_i] + \frac{h^6}{4132800}[11y^{(7)}_{i+1} - 15y^{(7)}_i],$$

and

$$6a_{i+1,3} = \frac{-1080}{41h^3}[y_{i+1} - y_i] + \frac{1080}{41h^2}a_{i,1} + \frac{402}{41}a_{i,3} + \frac{1}{41h}[154y''_{i+1} + 386y''_i] +$$

$$\frac{h}{246}[23y^{(4)}_{i+1} + 31y^{(4)}_i] - \frac{h^4}{103320}[13y^{(7)}_{i+1} - 14y^{(7)}_i].$$

Now the coefficient matrix of the above system of equations can be found, in the unknowns $a_{i,1}, a_{i,3}, a_{i+1,1}$ and $a_{i+1,3}$, i=1, 2,…,n-1 which is a non-singular matrix and hence all the coefficients are determined uniquely.

Theorem 1: Let $y(x)$ be the exact solution of the system (1) and we assuming that $y_i, i = 1,2,...,n-1$, be the numerical solution of (1), and $S(x)$ be a unique ninth spline function which is a solution of the problem (2). Then for $x \in [x_0, x_1]$, we have:

$$\left\| S_0^{(9-r)}(x) - y^{(9-r)}(x) \right\| \leq \begin{cases} A\, h^r w_9(f;h), r = 9, & \frac{49}{47232}h^r w_9(f;h), r = 8 \\[2mm] B\, h^r w_9(f;h), r = 7, & \frac{1823}{68880}h^r w_9(f;h), r = 6 \\[2mm] \frac{125}{1148}h^r w_9(f;h), r = 5, & \frac{893}{2296}h^r w_9(f;h), r = 4 \\[2mm] \frac{675}{574}h^r w_9(f;h), r = 3, & \frac{229}{82}h^r w_9(f;h), r = 2 \\[2mm] \frac{182}{41}h^r w_9(f;h), r = 1, & \frac{135}{41}h^r w_9(f;h), r = 0. \end{cases}$$

where $A = \dfrac{323324628002651}{57640752303423423488}$, $B = \dfrac{1551065516263567}{9223372036854775808}$

And where $W_9(f;h)$ denotes the modules of continuity of $y^{(9)}$.

Proof: Let $x \in [x_0, x_1]$, from equation (2.1) and by using Taylor's expansion formula we get

$$S_{0_i}^{(9)}(x) - y^{(9)}(x) = 362880a_{0,9} - y_i^{(9)}(x) .(4)$$

Using (2) and $a_{0,9}$, we obtain

$$\left| S_0^{(9)}(x) - y^{(9)}(x) \right| = \left| 362880a_{0,9} - y^{(9)}(x) \right| \leq \frac{135}{41}w_9(f;h) .(5)$$

By taking the eight derivatives and subtracting the function, using Taylor's series expansion on $y^{(9)}(x)$ about $x = x_1$,

we get $\left| S_0^{(8)}(x) - y^{(8)}(x) \right| = \left| 40320a_{0,8} + 362880(x - x_1)a_{0,9} - y^{(8)}(x_0) + (x - x_0)y^{(9)}(\alpha_1) \right|$

$\leq \frac{182}{41}hw_9(f;h)$ where $x_0 < \alpha_1 < x_1$.(6)

Also, from (2) and by taking the seventh derivatives, we get

$$S_0^{(7)}(x) - y^{(7)}(x) = y_0^{(7)} + 40320a_{0,8} + 181440h^2 a_{0,9} - y^{(7)}(x),$$

hence

$$\left| S_0^{(7)}(x) - y^{(7)}(x) \right| \leq \frac{229}{82}h^2 w_9(f;h) \qquad (7)$$

and by taking the sixth derivatives and fromthe Taylor's series expansion on $y^{(6)}(x)$ about $x = x_1$, we get:

$$\left| S_0^{(6)}(x) - y^{(6)}(x) \right| \le \frac{675}{574} h^3 \left| y^{(9)}(\beta_1) - y^{(9)}(\beta_2) \right| \le \frac{675}{574} h^3 w_9(f;h) \text{ where } x_0 < \beta_1, \beta_2 < x_1 \quad (8)$$

From equations (2) and substituting the coefficients, we get: $\left| S_0^{(5)}(x) - y^{(5)}(x) \right| \le \frac{893}{2296} h^4 w_9(f;h)$ (9)

By continuing this process, we have

$$\left| S_0^{(4)}(x) - y^{(4)}(x) \right| \le \frac{125}{1148} h^5 w_9(f;h), \left| S_0^{(3)}(x) - y^{(3)}(x) \right| \le \frac{1823}{68880} h^6 w_9(f;h),$$

$$\left| S_0''(x) - y''(x) \right| \le \frac{323324628002651}{57640752303423423488} h^7 w_9(f;h)$$

$$\left| S_0'(x) - y'(x) \right| \le \frac{49}{47232} h^8 w_9(f;h)$$

and $\left| S_0(x) - y(x) \right| \le \frac{1551065516263567}{9223372036854775808} h^9 w_9(f;h)$.

Lemma 1: Let $y(x)$ be the exact solution of the system of (1.1). Assume that $y_i, i = 1, 2, ..., n-1$, be the numerical solutions of (1), then $\left| e_{i,1} \right| \le c_i h^8 w_9(f;h)$ where

$$e_{i,1} = a_{i,1} - y_i', \quad (10)$$

and c_i depend on the numbers of intervals.

Proof: For $y(x) \in C^9[0,1]$ then using Taylor's expansion formula, we have:

$$y(x) = y(x_i) + (x - x_i)y'(x_i) + \frac{(x - x_i)^2}{2} y''(x_i) + ... + \frac{(x - x_i)^9}{5040} y^{(9)}(\theta_i),$$

where $x_i < \alpha_i < x_{i+1}$, and similar expressions for the derivatives of y(x) can be used.

Now if i=1 then from equation of $a_{1,1}$ and by using (10) we obtain

$$e_{1,1} = a_{1,1} - y_1' = \frac{h^8}{137760} y^{(9)}(\alpha_1) - \frac{h^8}{40320} y^{(9)}(\alpha_2) +$$

$$\frac{17h^8}{688800} y^{(9)}(\alpha_3) - \frac{h^8}{118080} y^{(9)}(\alpha_5) + \frac{11h^8}{8265600} y^{(9)}(\alpha_6) \quad (11)$$

where $x_0 < \alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6 < x_1$.

fromequation (11) we get :

$$\left| e_{1,1} \right| \le \frac{h^8}{330624} w_9(f;h) \rightarrow \left| e_{1,1} \right| \le h^8 c_1 w_9(f;h)$$

where $c_1 = \frac{1}{330624}$ Also, if i=2, then from the expression of $a_{2,1}$ and by using (10) we obtain

$$\left| e_{2,1} \right| \le c_2 h^8 w_9(f;h)$$

Continuing the same process, the inequality $\left| e_{i,1} \right| \le c_i h^8 w_9(f;h)$ for i= 1, ..., n-1 holds.

Lemma 2: Let $y(x)$ be the exact solution of the system of (1.1). Assume that $y_i, i = 1, 2, ..., n-1$, be the numerical solutions of (1), then $\left| e_{i,3} \right| \le c_i' h^6 w_9(f;h)$ for i= 1, ..., n-1

where $e_{i,3} = 6a_{i,3} - y_i'''$, $\quad (12)$

and c_i' depend on the numbers of intervals.

Proof: Let $y(x) \in C^9[0,1]$ and by using the Taylor's expansion formula and similar expressions for the derivatives of y (x), we have,if i=1 then from the expressions $a_{1,3}$ and from equation (12), we obtain

$$e_{1,3} = 6a_{1,3} - y_1''' = -\frac{h^6}{13776} y^{(9)}(\alpha_1) + \frac{11h^6}{14760} y^{(9)}(\alpha_2) -$$

$$\frac{h^6}{720} y^{(9)}(\alpha_3) + \frac{23h^6}{29520} y^{(9)}(\alpha_5) - \frac{13h^6}{206640} y^{(9)}(\alpha_6) \tag{13}$$

where $x_0 < \alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6 < x_1$.

and thus, we get: $|e_{1,3}| \leq \frac{h^6}{656} w_9(f;h)$ and $c_1' = \frac{1}{656}$

Also, if i=2 then from $a_{i,3}$ andequation (12), we get $|e_{2,3}| \leq c_2' h^6 w_9(f;h)$

Similarly the inequality $|e_{i,3}| \leq c_i' h^7 w_9(f;h)$ for i= 1, ..., n-1 holds.

Theorem 2: Let $S(x)$ be a unique spline function of ninth degreeand $y(x) \in C^9[0,1]$ be the solution of (2). Then for $x \in [x_i, x_{i+1}]$; i=1, 2,..., n-1, the following error bounds are holds:

$$\|S^{(r)}(x) - y^{(r)}(x)\| \leq \begin{cases} \frac{h^9}{a}(ac_i + bc_i' + c) w_9(f;h), & r = 0, \\ \frac{h^8}{47232}(47232 c_i + 23616 c_i' + 49) w_9(f;h), & r = 1, \\ (h^7 c_i' + \frac{3233246283002651}{576460752303423488}) w_9(f;h), & r = 2, \\ \frac{i}{68880} h^6(68880 c_i + 1823)w_9(f;h), & r = 3, \\ \frac{893i}{2296} h^5 w_9(f;h), r = 4, \frac{125i}{1148} h^4 w_9(f;h), r = 5, \frac{675i}{574} h^3 w_9(f;h), r = 6, \\ \frac{229i}{82} h^2 w_9(f;h), r = 7, \frac{182i}{41} h w_9(f;h), r = 8, \frac{135i}{41} w_9(f;h), r = 9. \end{cases}$$

$a = 3689348814741903232, b = 614891469126816900, c = 6204262065054268,$

Proof: Let $x \in [x_i, x_{i+1}]$ where i=1, 2, ..., n-1, then from equation (3) and using Taylor's expansion formula we get

$$S_i^{(9)}(x) - y^{(9)}(x) = 362880a_{i,9} - y^{(9)}(x) \tag{14}$$

Using (2) and $a_{0,9}$, we obtain

$$|S_i^{(9)}(x) - y^{(9)}(x)| = |362880a_{i,9} - y^{(9)}(x)| \leq \frac{135i}{41} w_9(f;h) \tag{15}$$

From (2) and by taking the eight derivatives and subtracting the function $y^{(8)}(x)$, andusing Taylor's series expansion on $y^{(8)}(x)$ about $x = x_i$, we get

$$|S_i^{(8)}(x) - y^{(8)}(x)| \leq \frac{182i}{41} hw_9(f;h) \text{ where } x_i < \alpha_{i,1} < x_i. \tag{16}$$

Also, from (3) and taking the seventh derivatives, we get .

$$|S_i^{(7)}(x) - y^{(7)}(x)| \leq \frac{229i}{82} h^2 w_9(f;h) \qquad \text{where } x_i < \alpha_{i,2} < x_i. \tag{17}$$

And by taking the sixth derivatives with using Taylor's series expansion on $y^{(6)}(x)$ about $x = x_i$, we get

$$\left|S_i^{(6)}(x) - y^{(6)}(x)\right| \le \frac{675i}{574} h^3 \left|y^{(9)}(\alpha_1) - y^{(9)}(\alpha_2)\right| \le \frac{675i}{574} h^3 w_9(f;h) \qquad (18)$$

where $x_i < \alpha_{i,3}, \alpha_{i,4} < x_{i+1}$, and also

$$\left|S_i^{(5)}(x) - y^{(5)}(x)\right| \le \frac{125i}{1148} h^4 w_9(f;h) \quad \text{where } x_i < \alpha_{i,3}, \alpha_{i,4}, \alpha_{i,5} < x_{i+1} \quad (19)$$

by the same process way we can find that: $\left|S_i^{(4)}(x) - y^{(4)}(x)\right| \le \frac{893i}{2296} h^5 w_9(f;h)$,

$$\left|S_i'''(x) - y'''\right| \le \frac{i h^6}{68880}(68880 c_i' + 1823) w_9(f;h)$$

Now using lemma (4) and (5), we obtain

$$\left|S_i''(x) - y''\right| \le h^7 i c_i' w_9(f;h) + \frac{3233246283002651 i}{576460752303423488} h^8 w_9(f;h)$$

$$\le (h^7 i c_i' + \frac{3233246283002651 i}{576460752303423488}) w_9(f;h)$$

$$\left|S_i'(x) - y'\right| \le \left|a_{i,1} - y_i'\right| + \frac{h^2}{2}\left|6 a_{i,3} - y_i''\right| + \left|5 h^4 a_{i,5} + 6 h^5 a_{i,6} + 8 h^7 a_{i,8} + 9 h^8 a_{i,9}\right|$$

$$\le \frac{h^8}{47232}(47232 c_i + 23616 c_i' + 49) w_9(f;h)$$

and

$$\left|S_i(x) - y(x)\right| \le h^9 (a_{i,1} - y_i') w_9(f;h) + \frac{h^9}{6}(6 a_{i,3} - y_i''') w_9(f;h) +$$

$$\frac{6204262065054268 h^9}{3689348814741903232} w_9(f;h) \le \frac{h^9}{a}(a c_i + b c_i' + c) w_9(f;h)$$

where

$$a = 3689348814741903232, b = 614891469126816900, c = 6204262065054268.$$

This proves **Theorem 2** for $x \in [x_i, x_{i+1}]$, $i = 1, 2,..., n-1$

NUMERICAL ILLUSTRATION

In this section, the nonlinear differential system of Magnetohidrodynamics free convection flow is presented and this problem is referred to[8] and [9]. The problems are tested to the efficiency of the development solutions, and to demonstrate its convergence computationally. The problems have been solved by using our method with different values of step size h; it's tabulated in Tables 1. These show that our results are more accurate.

Problem 1:[8] Consider the system

$$f''' + f f'' - (f')^2 + g_s \theta + g_c \varphi - M(f')^2 = 0$$

$$\theta'' - r \Pr f' \theta + \Pr f \theta' + \Pr Df \varphi'' = 0$$

$$\varphi'' - r Sc f' \varphi + Sc f \varphi' + Sc Sr \theta'' = 0 \qquad (20)$$

Where $f = f_w$, $f' = 1$, $\theta' = 1$, $\varphi' = 1$, at $\eta = 0$,

$f' = 0$, $\theta = 0$, $\varphi = 0$, at $\eta \to \infty$,

The following notations will indicate in the tables:

TOL	Tolerance
FS	Total failure steps

AMAXE   Absolute of the maximum errorwith respect to derivatives

and   TIME(ms)   The execution time taken in microseconds.

The absolute of maximum error with respect to derivatives defined as [4]:

$$\text{AMAXE}_{(f)}^{(i)} = \max_{1 \le i \le n}\left\|e_i^{(j)}\right\| = \max_{1 \le i \le n}\left\|s^{(j)}(x_i) - f^{(j)}(x_i)\right\| \quad \text{AMAXE}_{(\varphi)}^{(i)} = \max_{1 \le i \le n}\left\|e_i^{(j)}\right\| = \max_{1 \le i \le n}\left\|s^{(j)}(x_i) - \varphi^{(j)}(x_i)\right\|$$

$$\text{and} \quad \text{AMAXE}_{(\theta)}^{(i)} = \max_{1 \le i \le n}\left\|e_i^{(j)}\right\| = \max_{1 \le i \le n}\left\|s^{(j)}(x_i) - \theta^{(j)}(x_i)\right\|$$

Where j be the order of derivatives on whole intervals and $f(x_i)$, $\varphi(x_i)$ and $\theta(x_i)$ are the approximate solution of system (20),we can seefrom Taylors expansion that $S'(x_0) = f'(x_0)$ and $S'''(x_0) = f'''(x_0)$, $S'(x_0) = \varphi'(x_0)$ and $S'''(x_0) = \varphi'''(x_0)$ and $S'(x_0) = \theta'(x_0)$ and $S'''(x_0) = \theta'''(x_0)$.

Table 1: Absolute maximum error for $S(\eta)$ and its derivative with different values of tolerance for the problem1:

TOL	FS	$\text{AMAXE}_{(f)}^{(0)}$	$\text{AMAXE}_{(f)}^{(1)}$	$\text{AMAXE}_{(f)}^{(2)}$	$\text{AMAXE}_{(f)}^{(3)}$	TIME(ms)
10^{-1}	0	1.022×10^{-1}	20.397×10^{-1}	16.891×10^{-1}	1.701×10^{-1}	0.080082
10^{-2}	0	1×10^{-3}	2.378×10^{-1}	16.53×10^{-2}	17×10^{-3}	0.082031
10^{-3}	0	1×10^{-3}	2.41×10^{-3}	16.5×10^{-3}	17×10^{-4}	0.100690
TOL	FS	$\text{AMAXE}_{(f)}^{(4)}$	$\text{AMAXE}_{(f)}^{(5)}$	$\text{AMAXE}_{(f)}^{(6)}$	$\text{AMAXE}_{(f)}^{(7)}$	TIME(ms)
10^{-1}	0	32.91×10^{-1}	1.4577×10^{-2}	3.099×10^{0}	1.254×10^{-3}	0.080082
10^{-2}	0	3.086×10^{-1}	1.1947×10^{0}	23.309×10^{-1}	2.3481×10^{-4}	0.082031
10^{-3}	0	3.03×10^{-2}	13×10^{-2}	23.31×10^{-1}	2.456×10^{-6}	0.100690

Table 1(a): Errors Estimations between $S(\eta)$ and $f(\eta)$.

TOL	FS	$\text{AMAXE}_{(\varphi)}^{(0)}$	$\text{AMAXE}_{(\varphi)}^{(1)}$	$\text{AMAXE}_{(\varphi)}^{(2)}$	$\text{AMAXE}_{(\varphi)}^{(3)}$	TIME(ms)
10^{-1}	0	1.003×10^{-1}	1.539×10^{-1}	34×10^{-4}	4.86×10^{-2}	0.000218
10^{-2}	0	1×10^{-2}	15.3×10^{-3}	13×10^{-4}	49×10^{-4}	0.00020
10^{-3}	0	1×10^{-3}	15×10^{-4}	13.883×10^{-5}	49.138×10^{-5}	0.000214
TOL	FS	$\text{AMAXE}_{(\varphi)}^{(4)}$	$\text{AMAXE}_{(\varphi)}^{(5)}$	$\text{AMAXE}_{(\varphi)}^{(6)}$	$\text{AMAXE}_{(\varphi)}^{(7)}$	TIME(ms)
10^{-1}	0	61.47×10^{-2}	11.667×10^{-2}	3.7679×10^{0}	2.945×10^{-3}	0.000218
10^{-2}	0	5.65×10^{-2}	9.97×10^{-2}	3.768×10^{-1}	7.623×10^{-5}	0.00020
10^{-3}	0	56×10^{-4}	98×10^{-4}	3.77×10^{-2}	6.8301×10^{-7}	0.000214

Table 1(b): Errors Estimations between $S(\eta)$ and $\varphi(\eta)$.

TOL	FS	$\text{AMAXE}_{(\theta)}^{(0)}$	$\text{AMAXE}_{(\theta)}^{(1)}$	$\text{AMAXE}_{(\theta)}^{(2)}$	$\text{AMAXE}_{(\theta)}^{(3)}$	TIME(ms)

TOL	FS	$AMAXE^{(4)}_{(\theta)}$	$AMAXE^{(5)}_{(\theta)}$	$AMAXE^{(6)}_{(\theta)}$	$AMAXE^{(7)}_{(\theta)}$	TIME(ms)
10^{-1}	0	1.003×10^{-1}	8.505×10^{-1}	55×10^{-4}	28.69×10^{-2}	0.000204
10^{-2}	0	1×10^{-2}	8.53×10^{-3}	5×10^{-3}	2.87×10^{-2}	0.000205
10^{-3}	0	1×10^{-3}	8.5×10^{-3}	5.4907×10^{-4}	2.9×10^{-3}	0.000209
TOL	FS	$AMAXE^{(4)}_{(\theta)}$	$AMAXE^{(5)}_{(\theta)}$	$AMAXE^{(6)}_{(\theta)}$	$AMAXE^{(7)}_{(\theta)}$	TIME(ms)
10^{-1}	0	24.972×10^{-1}	11.667×10^{-2}	10.8899×10^{0}	0.1248×10^{-4}	0.000204
10^{-2}	0	2.436×10^{-1}	1.005×10^{-1}	1.089×10^{-1}	3.6102×10^{-5}	0.000205
10^{-3}	0	2.43×10^{-2}	9.6×10^{-3}	10.89×10^{-2}	4.456×10^{-7}	0.000209

Table 1(c):Errors Estimations between $S(\eta)$ and $\theta(\eta)$.

CONCLUSION

The approximate solutions of the nonlinear problems as differential system of Magnetohidrodynamics free convection flow by using ninth spline interpolation show that our method is better in the sense of accuracy and applicability. These have been verified by maximum absolute errors given in the tables it changes with respect to the step size of the tolerance and various problems. Some properties of spline are obtained which are required in proving the uniqueness, existences and convergence analysis of the present method.

RERERENCES

[1] Ahlberg, J. H.; Nilson, E. N. and Walsh, J. L. , 1967. The theory of splines and their applications, Academic Press, New York, London.

[2] Al Bayati, A. Y.; Saeed, R. K. and Hama-Salh, F. K. , 2009.The Existence, Uniqueness and Error Bounds of Approximation SplinesInterpolation for Solving Second-Order Initial Value Problems ,Journal ofMathematics and Statistics ,New York, 5,No.2,123-129.

[3] Eamail, M. N.; Fawzy, Th.; Ahmed, M. and Elmoselhi, H. O., 1994. Deficient spline function approximation to fourth order differential equations, Appl. Math. Modeling, 18, 658-664.

[4] Faraidun, K. H. , 2010. Numerical Solution for Fifth Order Initial Value Problems Using Lacunary Interpolation, Journal of Duhok University, Vol. 13, No. 1, pp.128-134.

[5] Jain M. K.; Iyengar, S.R.K. andJian R.K., 2007. Numerical Methods for Scientific and Engineering Computation, fifth edition, new Age International(P) Ltd.

[6] J. Hietarina, 1987. A Search for Bilinear Equations Passing Hirota's Three-Solition Condition, II mKdV-Type Bilinear Equations", Journal of Mathematical Physics, Vol. 28, No. 9, pp. 2094-2101.

[7] Lambert, J.D., 1991. Numerical Methods for Ordinary Differential Systems. ChichesterWiley.

[8] M. U. Ahammad, Md. ShirazulHoqueMollah, 2012. Numerical study of MHD free convection flow and mass transfer over a stretching sheet considering Dufour & Sort effects in the presence of Magnatic Field, Inter. J. Eng. & Tech. , IJET-IJENS, Vol:11 No: 5.

[9] O.K. Koriko, 2010. Approximate Solutions of a Higher Order MHD Flow of a Uniformly Stretched Vertical Permeable Surface in the Presence of Heat Generation/Absorption which Resulted from a Quadratic Reaction, Journal of Mathematics and Statistics 6 (2):96-99, ISSN 1549-3644.

[10] Rana, S. S. and Dubey, Y. P., 1999.Best error bounds for deficient quartic spline interpolation, Indian J. Pure Appl. Math., 30 (4), 385-393.

[11] Rostam K. Saeed, Faraidun K. Hamasalh, Gulnar W. Sadiq, 2012.Convergence of Ninth Spline Function to the Solution of a System of Initial Value Problems, World Applied Siences Journal, 16 (10),: 1360-1367.

[12] W. Herman and A. Nuseir, 1997, Symbolic Methods to Construct Exact Solution of Nonlinear Partial Differential Equations, Mathematics and Computers in Simulation, Vol. 43, pp.13-27.