



Continuous & Piecewise Convex Behavior Of Maximum Values Of Some Generalized Measures Of Fuzzy Cross Entropy.

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Abstract-- Consider two fuzzy sets A and B with same supporting points x_1, x_2, \dots, x_n and the corresponding fuzzy vectors $(\mu_A(x_1), \mu_A(x_2), \dots, \mu_A(x_n))$ and $(\mu_B(x_1), \mu_B(x_2), \dots, \mu_B(x_n))$ respectively, where each $\mu_A(x_i), 1 \leq i \leq n$ can vary subject to the total fuzziness $\sum_{i=1}^n \mu_A(x_i) = k$ and each $\mu_B(x_i)$ is known to us. In this paper, a comprehensive and straightforward procedure is made available to find the maximum values of some existing measures of generalized fuzzy cross entropy. Further, the continuous and piecewise convex behavior of the values so obtained is also discussed mathematically as well as graphically. Finally, a numerical example is used to illustrate the proposed method.

Keywords- Fuzzy set, fuzzy vector, fuzzy cross entropy, continuity piecewise convexity, maximum value.



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1. INTRODUCTION

In spite of the large diversity of mathematical models that are actually encountered in the domain of entropy optimization problems, Aczel[1] and others endorsed the view that generalized measures of cross-entropy should be introduced whenever the maximization of these measures leads to useful mathematical models in various fields. The optimization of generalized measures fuzzy cross entropy arise very naturally in response to the need to explain the existence of a variety of models in different contexts. As introduced by Zadeh[5]. Consider a given fuzzy set A with n supporting points (x_1, x_2, \dots, x_n) corresponding to the fuzzy vector $(\mu_A(x_1), \mu_A(x_2), \dots, \mu_A(x_n))$, where $\mu_A(x_i)$ is the degree of membership of the elements x_i of the set A . Our purpose is find the maximum values of some well known measures of generalized fuzzy cross entropy subject to the total fuzziness $\sum_{i=1}^n \mu_A(x_i) = k, 0 \leq k \leq n$. For this, we have considered some well known generalized measures of fuzzy cross entropy which are given below:

Corresponding to Renyi's [4] measure of fuzzy entropy, Kapur's [3] measure of fuzzy cross entropy is given by the following mathematical expression:

$$D_\alpha(A : B) = \frac{1}{\alpha - 1} \sum_{i=1}^n \log \left[\mu_A^\alpha(x_i) \mu_B^{1-\alpha}(x_i) + (1 - \mu_A(x_i))^\alpha (1 - \mu_B(x_i))^{1-\alpha} \right], \alpha \neq 1, \alpha > 0 \quad \dots(1)$$

Corresponding to Havrda and Charvet [2] measure of fuzzy entropy, Kapur's [3] measure of fuzzy cross entropy is given by the following mathematical expression:

$$D^\alpha(A : B) = \frac{1}{\alpha - 1} \sum_{i=1}^n \left[\mu_A^\alpha(x_i) \mu_B^{\alpha-1}(x_i) + (1 - \mu_A(x_i))^\alpha (1 - \mu_B(x_i))^{\alpha-1} \right], \alpha \neq 1, \alpha > 0 \quad \dots(2)$$

It has been proved that the maximum fuzzy cross entropy is a continuous and piecewise convex function of k . For illustration, a numerical example is used to show that the maximum cross entropy is monotonically decreasing when it is given that $\mu_B(x_i)$ is monotonically increasing for $1 \leq i \leq n$. Finally, a graph for maximum fuzzy cross entropy against the values of $0 \leq k \leq n$ is also drawn.

2. Maximum Value of $D_\alpha(A : B)$:

Since $D_\alpha(A : B)$ is a measure of generalized fuzzy entropy, its maximum value exists. Consider the following cases:

Case I : When k is any positive integer, say $k = m$, then we can choose m values of $\mu_A(x_i)$ as unity and remaining $(n - m)$ values as zero. i.e., $\mu_A(x_i) = (1, 1, 1, \dots, 1, 0, 0, \dots, 0)$. In this case, From equation (1), we have

$$D_\alpha(A : B) = \frac{1}{\alpha - 1} \left[\sum_{i=1}^m \log \left[\mu_A^\alpha(x_i) \mu_B^{1-\alpha}(x_i) + (1 - \mu_A(x_i))^\alpha (1 - \mu_B(x_i))^{1-\alpha} \right] + \sum_{i=m+1}^n \log \left[\mu_A^\alpha(x_i) \mu_B^{1-\alpha}(x_i) + (1 - \mu_A(x_i))^\alpha (1 - \mu_B(x_i))^{1-\alpha} \right] \right]$$

$$\therefore \text{Max.} D_\alpha(A : B) = - \left[\sum_{i=1}^m \log \mu_B(x_i) + \sum_{i=m+1}^n \log(1 - \mu_B(x_i)) \right] \quad \dots(3)$$

2.1 Illustration :

Suppose $\mu_B(x_i)$ is monotonically decreasing i.e., $\mu_B(x_i) = \left(\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \dots, \frac{1}{n+1} \right)$

(i) When $m = 0$, equation (3) gives

$$\text{Max.} D_\alpha(A : B) = - \sum_{i=1}^n \log(1 - \mu_B(x_i)) = - \left(\log \frac{1}{2} + \log \frac{2}{3} + \log \frac{3}{4} + \dots + \log \frac{n}{n+1} \right) = \log(n+1)$$

(ii) When $m = 1$, equation (3) gives

$$\text{Max.} D_\alpha(A : B) = - \left[\log \mu_B(x_1) + \sum_{i=2}^n \log(1 - \mu_B(x_i)) \right] = \log(n+1)$$

(iii) When $m = 2$, equation (3) gives

$$\text{Max.} D_\alpha(A : B) = - \left[\log \mu_B(x_1) + \log \mu_B(x_2) + \sum_{i=3}^n \log(1 - \mu_B(x_i)) \right] = \log 2 + \log(n+1)$$



(iv) When $m = 3$, equation (3) gives

$$Max.D_{\alpha}(A : B) = - \left[\log \mu_B(x_1) + \log \mu_B(x_2) + \sum_{i=3}^n \log(1 - \mu_B(x_i)) \right] = \log 2 + \log 3 + \log(n+1)$$

(v) When $m = 4$, equation (3) gives

$$Max.D_{\alpha}(A : B) = \log 2 + \log 3 + \log 4 + \log(n+1)$$

(vi) When $m = n$, equation (3) gives

$$Max.D_{\alpha}(A : B) = - \sum_{i=1}^n \log \mu_B(x_i) = - \left(\log \frac{1}{2} + \log \frac{1}{3} + \log \frac{1}{4} + \dots + \log \frac{1}{n+1} \right) = \log(n+1)$$

The graph of $Max.D_{\alpha}(A : B)$ against the values $0 \leq k \leq n$ is shown in the following figure 1:

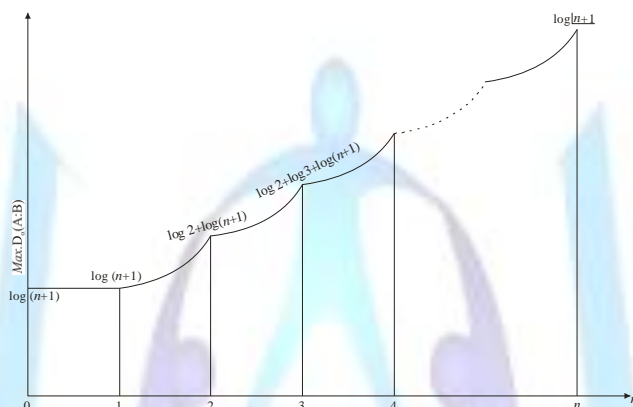


Figure 1: Maximum values of $D_{\alpha}(A : B)$ against the values $0 \leq k \leq n$

From figure 1, we conclude that $Max.D_{\alpha}(A : B)$ is monotonically increasing when $\mu_B(x_i)$ is monotonically decreasing. Also, the $Max.D_{\alpha}(A : B)$ is a piecewise convex function of k .

Case II : When $k = m + \xi$, where m is any non-negative integer and ξ is a positive fraction. We put m values of $\mu_A(x_i)$ as unity, $(m+1)^{th}$ values as ξ and remaining $(n - m - 1)$ values as zero i.e., $\mu_A(x_i) = (1, 1, 1, \dots, 1, \xi, 0, 0, \dots, 0)$.

In this case, Using equation (1), we have

$$D_{\alpha}(A : B) = \frac{1}{\alpha - 1} \left[\sum_{i=1}^m \log \left[\mu_A^{\alpha}(x_i) \mu_B^{1-\alpha}(x_i) + (1 - \mu_A(x_i))^{\alpha} (1 - \mu_B(x_i))^{1-\alpha} \right] + \log \left[\mu_A^{\alpha}(x_{m+1}) \mu_B^{1-\alpha}(x_{m+1}) + (1 - \mu_A(x_{m+1}))^{\alpha} (1 - \mu_B(x_{m+1}))^{1-\alpha} \right] + \sum_{i=m+2}^n \log \left[\mu_A^{\alpha}(x_i) \mu_B^{1-\alpha}(x_i) + (1 - \mu_A(x_i))^{\alpha} (1 - \mu_B(x_i))^{1-\alpha} \right] \right]$$

$$\therefore Max.D_{\alpha}(A : B) = - \sum_{i=1}^m \log \mu_B(x_i) + \frac{1}{\alpha - 1} \log \left[\xi^{\alpha} \mu_B^{1-\alpha}(x_{m+1}) + (1 - \xi)^{\alpha} (1 - \mu_B(x_{m+1}))^{1-\alpha} \right] - \sum_{i=m+2}^n \log(1 - \mu_B(x_i)) \quad \dots(4)$$

2.2 Illustration :

Suppose $\mu_B(x_i)$ is monotonically decreasing, so we take

$$\mu_B(x_i) = \left(\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \dots, \frac{1}{m+1}, \frac{1}{m+2}, \frac{1}{m+3}, \frac{1}{m+4}, \dots, \frac{1}{n+1} \right)$$

Using, equation (1), we have

$$Max.D_{\alpha}(A : B) = \log 2 + \log 3 + \log 4 + \dots + \log(m+1) + \log \frac{m+3}{m+2} + \log \frac{m+4}{m+3} + \dots + \log \frac{n+1}{n} + \frac{1}{\alpha - 1} \log \left[\xi^{\alpha} (m+2)^{\alpha-1} + (1 - \xi)^{\alpha} \left(\frac{m+2}{m+1} \right)^{\alpha-1} \right] \quad \dots(5)$$

$= \log(n+1)m + \phi(\xi)$, where

$$\phi(\xi) = \frac{1}{\alpha - 1} \log \left(\xi^{\alpha} (m+1)^{\alpha-1} + (1 - \xi)^{\alpha} \right) \quad \dots(6)$$



2.3 Convexity of $\phi(\xi)$:

Take $h(\xi) = \xi^\alpha (m+1)^{\alpha-1} + (1-\xi)^\alpha$. Differentiating it w.r.t. ξ , we get

$$h'(\xi) = \alpha(\xi^{\alpha-1}(m+1)^{\alpha-1} - (1-\xi)^{\alpha-1}); h''(\xi) = (\alpha^2 - \alpha)(\xi^{\alpha-2}(m+1)^{\alpha-1} + (1-\xi)^{\alpha-2})$$

- (i) If $\alpha > 1$, then $h''(\xi) > 0 \Rightarrow h(\xi)$ is a convex function of $\xi \Rightarrow \log h(\xi)$ is a convex function of $\xi \Rightarrow \frac{1}{\alpha-1} \log h(\xi)$ is a convex function of ξ for each $\alpha > 1$.
- (ii) If $0 < \alpha < 1$, then $h''(\xi) < 0 \Rightarrow h(\xi)$ is a concave function of $\xi \Rightarrow \log h(\xi)$ is a concave function of $\xi \Rightarrow \frac{1}{\alpha-1} \log h(\xi)$ is a convex function of ξ for each $0 < \alpha < 1$

Thus, $\phi(\xi)$ is a convex function of ξ for each α . So, its minimum value exists. For minimum, we put

$$\frac{d}{d\xi} \phi(\xi) = 0, \text{ which gives } \frac{\alpha}{\alpha-1} \frac{\xi^{\alpha-1}(m+1)^{\alpha-1} - (1-\xi)^{\alpha-1}}{\xi^\alpha (m+1)^{\alpha-1} + (1-\xi)^\alpha} = 0 \quad | \text{ Using equation (6)}$$

$$\Rightarrow \xi(m+2) - 1 = 0 \Rightarrow \xi = \frac{1}{m+2}$$

Therefore, the minimum value of $\phi(\xi)$ exists at $\xi = \frac{1}{m+2}$. Hence, from equation (8), the minimum value of $Max.D_\alpha(A : B)$ is :

$$\begin{aligned} Min.Max.D_\alpha(A : B) &= \log \lfloor m(n+1) \rfloor + \phi\left(\frac{1}{m+2}\right) = \log \lfloor m(n+1) \rfloor + \frac{1}{\alpha-1} \log \left(\frac{(m+1)^{\alpha-1}}{(m+2)^\alpha} + \left(\frac{m+1}{m+2}\right)^\alpha \right) \\ &= \log \frac{(n+1) \lfloor m(m+1) \rfloor}{m+2} = \log \frac{(n+1) \lfloor m+1 \rfloor}{m+2} = \log(n+1) + \log \frac{\lfloor m+1 \rfloor}{m+2} \end{aligned}$$

For different values of m , the different values of $Min.Max.D_\alpha(A : B)$ is shown in the following table 1

Value of m	Minimum value of $Min.Max.D_\alpha(A : B)$
$m = 0$	$\log \frac{(n+1) \lfloor 1 \rfloor}{2}$
$m = 1$	$\log \frac{(n+1) \lfloor 2 \rfloor}{3}$
$m = 2$	$\log \frac{(n+1) \lfloor 3 \rfloor}{4}$
$m = 3$	$\log \frac{(n+1) \lfloor 4 \rfloor}{5}$
$m = n-1$	$\log \frac{(n+1) \lfloor n \rfloor}{n+1}$
$m = n$	$\log \frac{(n+1) \lfloor n+1 \rfloor}{n+2}$

Table 1 : Minimum values of $Max.D_\alpha(A : B)$ against the values $0 \leq m \leq n$.

From table1, we can plot the graph of $Min.Max.D_\alpha(A : B)$ against the values of $0 \leq m \leq n$ as shown in the following figure 2 :

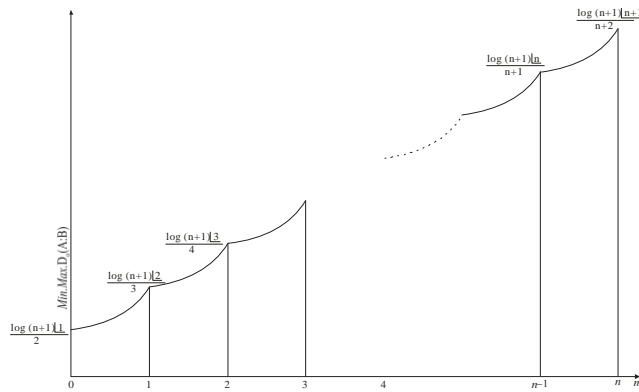


Figure : 2 Minimum value of $Max.D_\alpha(A:B)$ against the values $0 \leq m \leq n$

From figure 2, we conclude that when $\mu_B(x_i)$ is monotonically decreasing, $Max.D_\alpha(A:B)$ is monotonically increasing. Also $Min.Max.D^\alpha(A:B)$ is piecewise convex function of k .

3. Maximum Value of $D^\alpha(A:B)$:

Consider the following cases:

Case I : When k is any positive integer, say $k = m$, then, we can choose m values of $\mu_A(x_i)$ as unity and remaining $n - m$ values as zero i.e., $\mu_A(x_i) = (1, 1, 1, \dots, 1, 0, 0, \dots, 0)$

In this case, from equation (2), we have

$$D^\alpha(A:B) = \frac{1}{\alpha-1} \left[\sum_{i=1}^m \mu_A^\alpha(x_i) \mu_B^{1-\alpha}(x_i) + (1 - \mu_A(x_i))^\alpha (1 - \mu_B(x_i))^{1-\alpha} - 1 + \sum_{i=m+1}^n \left[\mu_A^\alpha(x_i) \mu_B^{1-\alpha}(x_i) + (1 - \mu_A(x_i))^\alpha (1 - \mu_B(x_i))^{1-\alpha} - 1 \right] \right]$$

$$\therefore Max.D^\alpha(A:B) = \frac{1}{\alpha-1} \left[\sum_{i=1}^m (\mu_B^{1-\alpha}(x_i) - 1) + \sum_{i=m+1}^n ((1 - \mu_B(x_i))^{1-\alpha} - 1) \right]$$

...(7)

3.1 Illustration :

Suppose $\mu_B(x_i)$ is monotonically decreasing i.e., $\mu_B(x_i) = \left(\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \dots, \frac{1}{n+1}\right)$

(i) When $m = 0$, equation (7) gives

$$Max.D^\alpha(A:B) = \frac{1}{\alpha-1} \left[\sum_{i=1}^n [(1 - \mu_B(x_i))^{1-\alpha} - 1] \right] = \frac{1}{\alpha-1} \left[2^{\alpha-1} + \left(\frac{3}{2}\right)^{\alpha-1} + \left(\frac{4}{3}\right)^{\alpha-1} + \dots + \left(\frac{n+1}{n}\right)^{\alpha-1} - n \right]$$

(ii) When $m = 1$, equation (7) gives

$$Max.D^\alpha(A:B) = \frac{1}{\alpha-1} \left[(\mu_B^{1-\alpha}(x_1) - 1) + \sum_{i=2}^n [(1 - \mu_B(x_i))^{1-\alpha} - 1] \right] = \frac{1}{\alpha-1} \left[2^{\alpha-1} + \left(\frac{3}{2}\right)^{\alpha-1} + \left(\frac{4}{3}\right)^{\alpha-1} + \dots + \left(\frac{n+1}{n}\right)^{\alpha-1} - n \right]$$

(iii) When $m = 2$, equation (7) gives

$$\begin{aligned} Max.D^\alpha(A:B) &= \frac{1}{\alpha-1} \left\{ (\mu_B^{1-\alpha}(x_1) - 1) + (\mu_B^{1-\alpha}(x_2) - 1) + \sum_{i=3}^n [(1 - \mu_B(x_i))^{1-\alpha} - 1] \right\} \\ &= \frac{1}{\alpha-1} \left[2^{\alpha-1} + 3^{\alpha-1} + \left(\frac{4}{3}\right)^{\alpha-1} + \left(\frac{5}{4}\right)^{\alpha-1} + \dots + \left(\frac{n+1}{n}\right)^{\alpha-1} - n \right] \end{aligned}$$

(iv) When $m = 3$, equation (7) gives

$$Max.D^\alpha(A:B) = \frac{1}{\alpha-1} \left[2^{\alpha-1} + 3^{\alpha-1} + 4^{\alpha-1} + \left(\frac{5}{4}\right)^{\alpha-1} + \left(\frac{6}{5}\right)^{\alpha-1} + \dots + \left(\left(\frac{n+1}{n}\right)^{\alpha-1} - n\right) \right]$$



(v) When $m = n$, equation (7) gives

$$Max.D^\alpha(A : B) = \frac{1}{\alpha - 1} \left[2^{\alpha-1} + 3^{\alpha-1} + 4^{\alpha-1} + \dots + (n+1)^{\alpha-1} - n \right]$$

The graph of $Max.D^\alpha(A : B)$ against the values $0 \leq k \leq n$ is shown in the following figure 3:

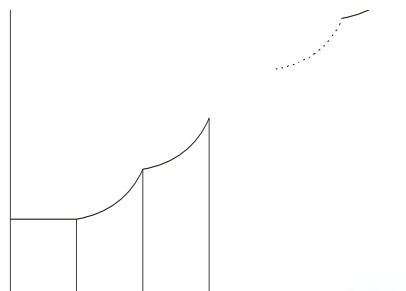


Figure 3: Maximum value of $D^\alpha(A : B)$ against the values $0 \leq k \leq n$

From figure 3, we conclude that $Max.D^\alpha(A : B)$ is monotonically increasing when $\mu_B(x_i)$ is monotonically decreasing. Also, the $Max.D^\alpha(A : B)$ is a piecewise convex function of k .

Case II. When $k = m + \xi$, where m is any non-negative integer and ξ is a positive fraction. We put m values of $\mu_A(x_i)$ as unity, $(m+1)^{th}$ value as ξ and remaining $(n - m - 1)$ values as zero i.e., $\mu_A(x_i) = (1, 1, 1, \dots, 1, \xi, 0, 0, \dots, 0)$. Using equation (2), we have

$$D^\alpha(A : B) = \frac{1}{\alpha - 1} \left[\sum_{i=1}^m \left[\mu_A^\alpha(x_i) \mu_B^{1-\alpha}(x_i) + (1 - \mu_A(x_i))^\alpha (1 - \mu_B(x_i))^{1-\alpha} - 1 \right] + \mu_A^\alpha(x_{m+1}) \mu_B^{1-\alpha}(x_{m+1}) + (1 - \mu_A(x_{m+1}))^\alpha (1 - \mu_B(x_{m+1}))^{1-\alpha} - 1 \right] \dots$$

$$+ \sum_{i=m+2}^n \left[\mu_A^\alpha(x_i) \mu_B^{1-\alpha}(x_i) + (1 - \mu_A(x_i))^\alpha (1 - \mu_B(x_i))^{1-\alpha} - 1 \right]$$

$$Max.D^\alpha(A : B) = \frac{1}{\alpha - 1} \left[\sum_{i=1}^m (\mu_B^{1-\alpha}(x_i) - 1) + \xi^\alpha \mu_B^{1-\alpha}(x_{m+1}) + (1 - \xi)^\alpha (1 - \mu_B(x_{m+1}))^{1-\alpha} - 1 + \sum_{i=m+2}^n [(1 - \mu_B(x_i))^{1-\alpha} - 1] \right] \dots(8)$$

3.2 Illustration :

Suppose $\mu_B(x_i)$ is monotonically decreasing i.e.; $\mu_B(x_i) = \left(\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \dots, \frac{1}{m+1}, \frac{1}{m+2}, \frac{1}{m+3}, \dots, \frac{1}{n+1} \right)$, from equation (2), we have

$$Max.D^\alpha(A : B) = \frac{1}{\alpha - 1} \left(2^{\alpha-1} + 3^{\alpha-1} + \dots + (m+1)^{\alpha-1} \right) + \frac{1}{\alpha - 1} \left[\left(\frac{m+3}{m+2} \right)^{\alpha-1} + \left(\frac{m+4}{m+3} \right)^{\alpha-1} + \dots + \left(\frac{n+1}{n} \right)^{\alpha-1} - n \right]$$

$$+ \phi(\xi), \text{ where } \phi(\xi) = \frac{1}{\alpha - 1} \left(\xi^\alpha (m+2)^{\alpha-1} + (1 - \xi)^\alpha \left(\frac{m+2}{m+1} \right)^{\alpha-1} \right)$$

3.3 Convexity of $\phi(\xi)$:

It can be easily proved that the function $\phi(\xi)$ is a convex function of ξ for each value of α . Hence the minimum value exists and it exists at $\xi = \frac{1}{m+2}$. Further,

$$Min.Max.D^\alpha(A : B) = \frac{1}{\alpha - 1} \left[2^{\alpha-1} + 3^{\alpha-1} + 4^{\alpha-1} + \dots + (m+1)^{\alpha-1} + \left(\frac{m+3}{m+2} \right)^{\alpha-1} + \left(\frac{m+4}{m+3} \right)^{\alpha-1} + \dots + \left(\frac{n+1}{n} \right)^{\alpha-1} - n \right] + \frac{1}{\alpha - 1} \dots(9)$$

(i) When $m = 0$, equation (9) gives

$$Min.Max.D^\alpha(A : B) = \frac{1}{\alpha - 1} \left[1 + \left(\frac{3}{2} \right)^{\alpha-1} + \left(\frac{4}{3} \right)^{\alpha-1} + \dots + \left(\frac{n+1}{n} \right)^{\alpha-1} - n + 1 \right] = g_0(\alpha), \text{ say}$$

(ii) When $m = 1$, equation (9) gives

$$Min.Max.D^\alpha(A : B) = \frac{1}{\alpha - 1} \left[2^{\alpha-1} + \left(\frac{4}{3} \right)^{\alpha-1} + \left(\frac{5}{4} \right)^{\alpha-1} + \dots + \left(\frac{n+1}{n} \right)^{\alpha-1} - n + 1 \right] = g_1(\alpha), \text{ say}$$

(iii) When $m = 2$, equation (9) gives

$$\text{Min.Max.}D^\alpha(A : B) = \frac{1}{\alpha - 1} \left[2^{\alpha-1} + 3^{\alpha-1} + \left(\frac{5}{4}\right)^{\alpha-1} + \left(\frac{6}{5}\right)^{\alpha-1} + \dots + \left(\frac{n+1}{n}\right)^{\alpha-1} - n + 1 \right] = g_2(\alpha), \text{ say}$$

(iv) When $m = 3$, equation (9) gives

$$\text{Min.Max.}D^\alpha(A : B) = \frac{1}{\alpha - 1} \left[2^{\alpha-1} + 3^{\alpha-1} + 4^{\alpha-1} + \left(\frac{6}{5}\right)^{\alpha-1} + \left(\frac{7}{6}\right)^{\alpha-1} + \dots + \left(\frac{n+1}{n}\right)^{\alpha-1} - n + 1 \right] = g_3(\alpha), \text{ say}$$

(v) When $m = n$, equation (9) gives

$$\text{Min.Max.}D^\alpha(A : B) = \frac{1}{\alpha - 1} \left[2^{\alpha-1} + 3^{\alpha-1} + 4^{\alpha-1} + \dots + (n+1)^{\alpha-1} - n + 1 \right] = g_n(\alpha), \text{ say}$$

The graph of $\text{Min.Max.}D^\alpha(A : B)$ against the values of $0 \leq m \leq n$ is shown in the following figure 4:

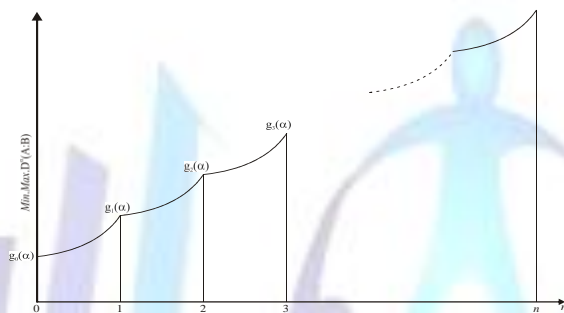


Figure 4 : Minimum value of $\text{Max.}D^\alpha(A : B)$ against the values $0 \leq k \leq n$

From figure 4, we conclude that $\text{Max.}D^\alpha(A : B)$ is monotonically increasing when $\mu_B(x_i)$ is monotonically decreasing. Also $\text{Max.}D^\alpha(A : B)$ is a piecewise convex function of k .

Conclusion : It has been proved that the maximum generalized fuzzy cross entropy is a continuous and piecewise convex function of k . Also, the maximum generalized fuzzy cross entropy is monotonically decreasing when it is given that $\mu_B(x_i)$ is monotonically increasing.,

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