Relative Study of Solvers for Finite Element Analysis

Harpal Singh
Guru Nanak DevEngineering College Ludhiana
hps_bhoday@yahoo.com

ABSTRACT

A relative study of various linear simultaneous equation solvers has been made. The profile solver and the block solver which have been considered to be most effective solution techniques have been implemented. Medium to large sized structural analysis problems have been analyzed. The comparison of solution time, total execution time and the incore memory requirement has been made. The suitability of solvers for various sized problems has also been discussed.

INTRODUCTION

It has been seen that for larger problems common in practice, 20 to 50 percent of the computer execution time may be devoted to solution of set of linear simultaneous equations. This figure may rise further to about 80 percent in dynamic, non-linear, or structural optimization problems. Therefore if inappropriate techniques are used for the solution of equilibrium equations, the total cost of analysis may be many times, up to 100 times larger than otherwise necessary. ANSYS [1] uses a wave front (or frontal) solution procedure for system of simultaneous linear equations developed from the assembled finite elements. The amount of core storage places a limit on size of front. WILSON et al. [14] proposed an out of core band solver, in which matrix is reduced by column instead of rows, so that most of the operations involving zero multipliers can be avoided. BATHE et al. [4] developed SAP-IV, a `Structural Analysis Program' for static and dynamic response of Linear system, which uses SESOL, a band solver for system of simultaneous linear equations. The concept of blocking is also used in this subroutine to increase the capacity of the solver. BATHE [2] used profile solver in the program ADINA. BATHE [3] presented `COLSOL', a subroutine for active column solution (or skyline solver) with full description, based on LDLT decomposition of stiffness matrix stored in a column IRONS [7] developed `A Frontal Solution Program' in which the complete stiffness matrix is never found but assembly is done element by element and elimination is also done side by side. HOOD [6] developed `A Frontal Solution Program for Unsymmetrical Equations'. MEYER [10] discussed the concept of blocking in detail to increase the capacity of band solvers. NATHAN and WILLIAM [11], developed a solution procedure for solution of linear equations for small computer systems, which is similar to frontal solution but here the equations are assembled node-by-node instead of element by element. SLOAN [12] developed a program for reducing the profile and wave front of a sparse matrix with a symmetric structure and it can be employed to derive efficient ordering for both profile and frontal solution schemes. ZIENKIEWICZ [15] presented package called ACTSOL and UACTCL, profile equation solving subroutines for symmetric and unsymmetrical equations respectively. The LU decomposition of stiffness matrix is performed in these two subroutines. McGUIRE and GALLACHER [13] discussed banded solver, concept of blocking and frontal solvers in detail with examples. MEEK, J.L. [9], presented subroutine to solve Banded symmetric matrix by Cholesky (LL^T) decomposition.

SOLUTION TECHNIQUES

The profile solver and the frontal solver have been considered to be the most effective solution techniques. These solvers have been implemented and described below.

Skyline (or Profile) Solver: This is based on Gauss-Elimination process. Here LDL^T decomposition of matrix is carried out. The only difference is in the method of storage of the matrix. For this decomposition can be modified accordingly. An effective storage scheme for the structure stiffness matrix is to store only the portion of each matrix column between the skyline (or profile) and the diagonal as shown in Fig. (1), in a one dimensional array. The matrix of Fig. (1) can be written in single subscript as shown in Fig. (2). Further, the different node numbering schemes can have different profile as shown in Fig. (3) and we should choose that profile which has minimum number of zero within profile because these zero can again create `fills'. The skyline solver is column-oriented and exploits the differing heights above the diagonal exhibited by various columns, as in many finite element analyses. The advantage of skyline storage scheme is that the storage requirements are not severely affected by a few very long columns, as shown in Fig. (1). That is why, the solution technique based on this storage scheme is also known as active column solver. One thing should be kept in mind, that node numbering which give least bandwidth will result in most compacted matrix A. Solution algorithm which uses such

International Journal of Computers & Technology
Volume 4 No. 2, March-April, 2013, ISSN 2277-3061

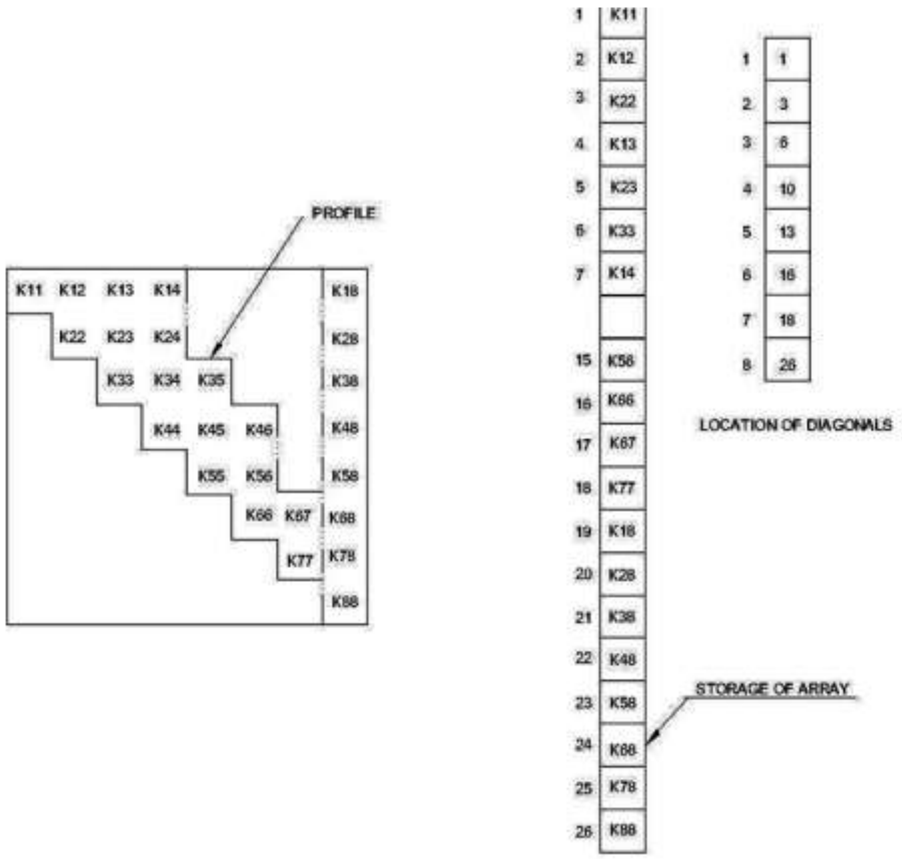


Fig. 1 An Effective Storage to Store Portion of Matrix Column Between Skyline.

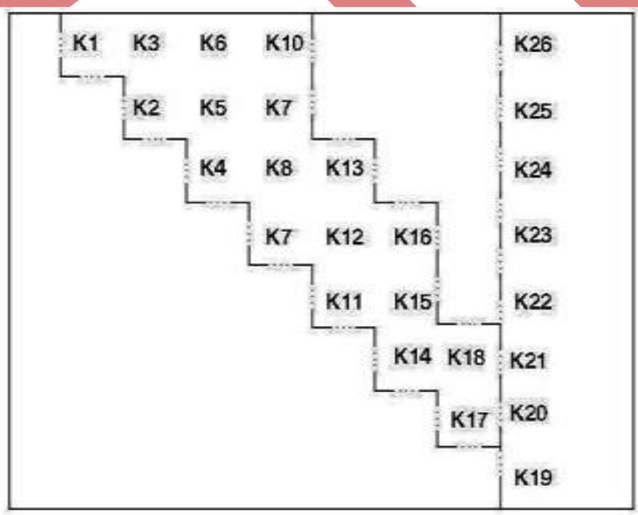


Fig. 2 Matrix of Fig. 1 Written in single Subscript Form.

storage scheme is known as profile solver [2, 18]. The decomposition of A into L D L^T is carried out in the usual way taking into account of its special storage system. The solution of equations is then carried out in usual manner.

Block Solver: These are also called `large-capacity band solvers'. These types of large capacity band solvers incorporate the concept of `blocking' as discussed in Ref. [4, 14]. Block storage and block processing can be described briefly by Fig. (3). Blocking is useful when the system is too large to be solved directly in the high speed core of the computer being used. In this event, the data can be stored in blocks in low speed (or back-up) storage as indicated schematically in Fig. 3(c). Each row in this schematic store contains the data from one equation. The size of each block may be governed either by nature of problem or by the characteristic of the computer. Storage economy is immediately apparent, since no provision need be made for elements outside of the populated band.

International Journal of Computers & Technology
Volume 4 No. 2, March-April, 2013, ISSN 2277-3061

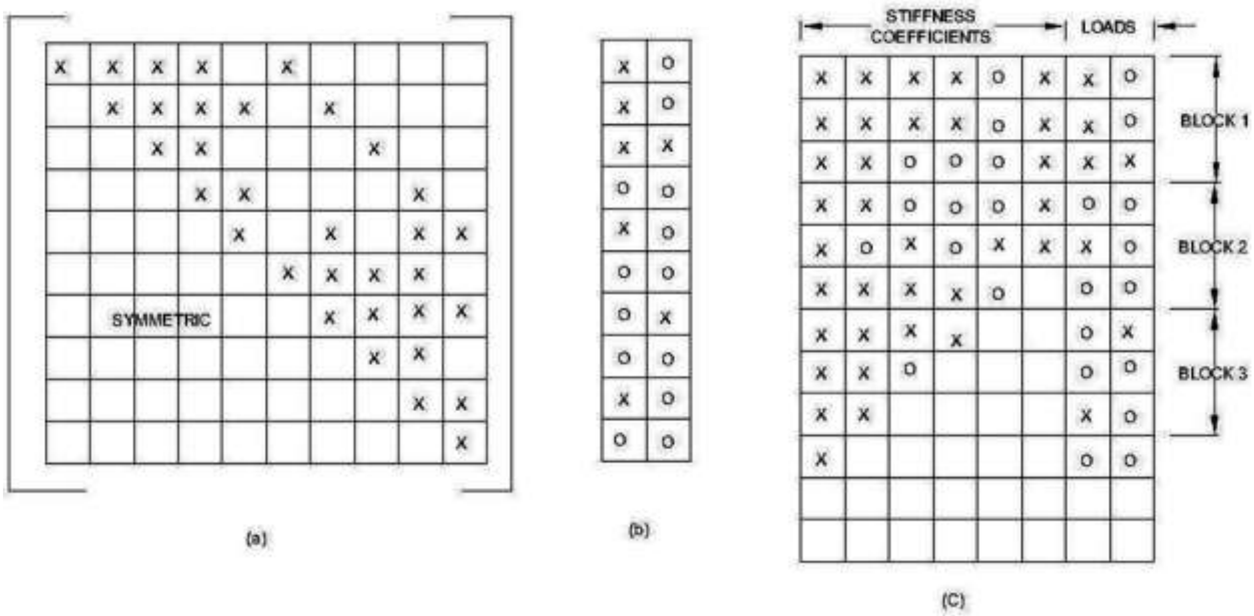


Fig. 3. Block Storage of Stiffness Coefficients and Loads (a) Stiffness Matrix (b) Load Vector (c) Block Storage

Table : 1 Blocking Done For Test Problem - 1

Solver	MTOT	NEQ	MBAND	NSKY	NEQB	NBLOCK
Block Solver	50,000	528	486	43932	47	12
	40,000	528	486	43932	37	15
	30,000	528	486	43932	27	20
	20,000	528	486	43932	16	33
	10,000	528	486	43932	6	88
	6,000	528	486	43932	2	264
	5,000	528	486	43932	1	528

DEVELOPMENT OF COMPUTER PROGRAM

A Computer program BSTAP has been developed which uses a Block solver subroutine SESOL [14] for direct solution of large systems of linear equations. A general purpose computer program PASSFEM [8] which uses the profile solver has been taken for the comparison of the solvers. The available high speed core (MTOT) is used in an optimum way by calculating the maximum block size possible.

DISCUSSION OF RESULTS

For the relative study of solvers and to check the suitability of the solvers for different problems, the general purpose computer programs PASSFEM [8] and present program BSTAP have been compared. The 486 DX computer has been used which has the following configuration :

Main Processor : 486 DX, Total RAM : 3968 KB

Base Memory Size : 640 KB , Ext. Memory Size : 3328 KB

Cache Memory Size : 128 KB, Frequency of CPU Clock : 33 MHz

MS FORTRAN 77 Compiler ver. 6.0

Following problems have been taken for the relative study.

Table : 2 Time Analysis Of Test Problem - 1

Solver	MTOT	Time taken for Decom-position {K}{P} (sec.)	Back substi-tution time (sec.)	Total Solution time (sec.)	Decom-position time as % of total solution time	Total analysis time (sec.)	Total solution time as % of total analysis time	Remark
	<47071	-	-	-	-	-	-	No solu-tion was possible due to less memory
Skyline solver	47071	23.62	1.32	24.64	94.70	914.73	2.73	-
	50000	23.89	1.32	25.21	94.22	906.33	2.78	-
Block solver	50000	77	76	153	50	243	63	-
	40000	89	75	164	54	277	59	-
	30000	104	73	178	58	313	56	-
	20000	157	88	245	63	461	53	-
	10000	439	159	598	73	1129	52	-
	6000	2058	352	2411	85	3979	60	-
	5000	-	-	-	-	-	-	No sol-ution as NEQB = 1 This is limit-ation of solver)

Test Problem-I : The unsymmetrical framed structure shown in Fig. (4) subjected to both lateral as well as vertical loads (UDL on all beams) has been taken, with 112 structure nodes and 208 elements in the structure and has been solved by both the programs PASSFEM and BSTAP in DOUBLE PRECISION. This test problem has been taken to compare the solution time and analysis time taken.

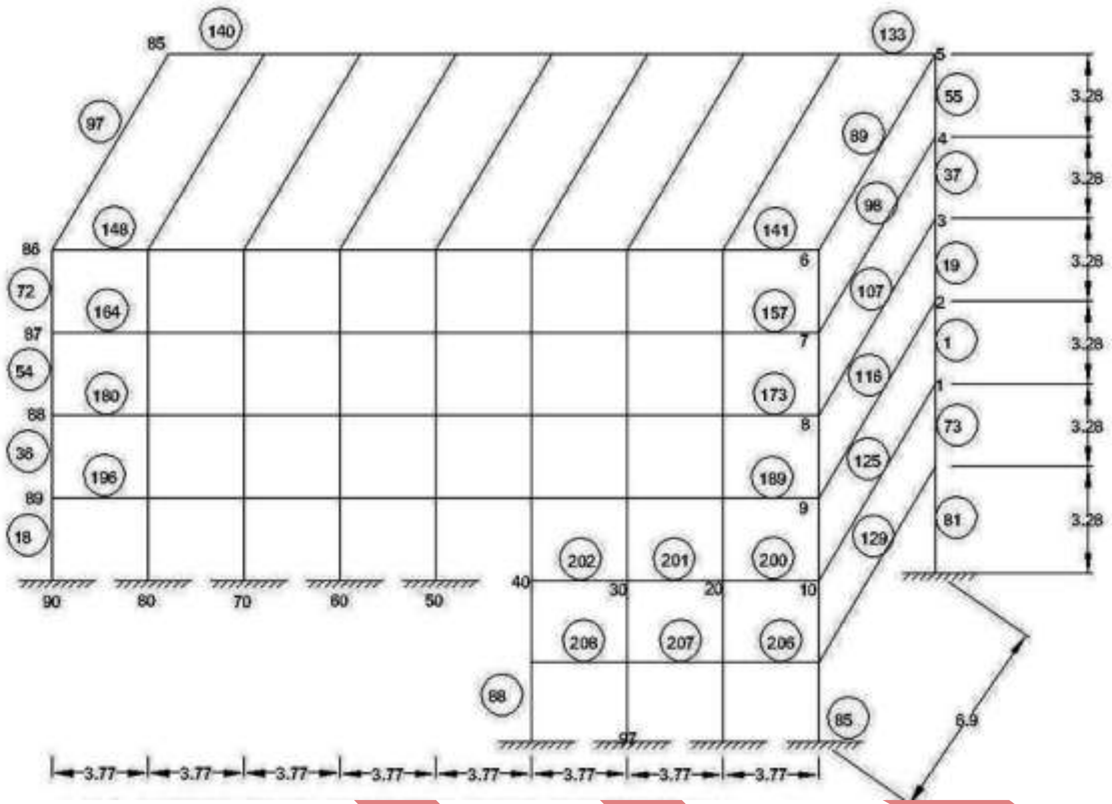


Fig. 4 Unsymmetrical Building Frame for Test Problem 1.

Test Problem-II : A 19 storey (4 bay x 5 bay) symmetrical framed structure shown in Fig. (6) subjected to both lateral as well as vertical loads (UDL on all beams) has been taken as a large sized problem to check the suitability of computer program BSTAP for large size problem having 600 structures nodes and 1501 elements in structure. The following results are obtained :

Number of structural degrees of freedom, NEQ : 3420

Bandwidth, MBAND : 186

Number of elements below skyline, NSKY : 5,99,814

NEQ*MBAND : 6,36,120

Table : 3 Blocking Done For Test Problem - I1

Solver	MTOT	NEQ	MBAND	NSKY	NEQB	NBLOCK
Block Solver	99,000	3420	186	599814	212	17
	80,000	3420	186	599814	161	22
	50,000	3420	186	599814	81	43
	45,000	3420	186	599814	68	51
	30,000	3420	186	599814	28	123

The results of Test Problem-II are given in Table 3 and 4 to see the suitability of block solver computer analysis program BSTAP for large sized problems, which otherwise can never be solved by PASSFEM within the configuration of the system discussed above. In Table 3 the blocking done by BSTAP depending upon high speed storage array MTOT is given and in Table 4 the overall analysis time, solution time are given for analysis both in Double Precision and Single Precision.

Total analysis time is 10 to 15 percent more in DOUBLE PRECISION than in SINGLE PRECISION. The variation of the solution time required with respect to NEQB and NEQ*MBAND has been shown for both the problems in Figs 7 and 8 respectively

Table : 4 Time Analysis Of Test Problem - I1

	MTOT	Preci-sion	Time taken for Decom-position {K}{P} (sec.)	Solver	Total Solution time (sec.)	Decom-position time as % of total solution time	Total analysis time (sec.)	Total solution time as % of total analysis time
Block solver	99000	SINGLE	624	101	725	86.5	1450	49.97
	80000	SINGLE	637	100	737	86.4	1614	45.69
	50000	SINGLE	652	99	751	86.0	2443	31.28
	45000	SINGLE	657	100	757	86.0	2742	27.37
	45000	DOUBLE	740	189	929	79.68	3152*	29.45
	30000	SINGLE	736	102	838	86.68	5560	15.08
	30000	DOUBLE	863	303	1166	73.99	6395*	18.20

The following observations have been made :

a. It has been observed that if we use BSTAP in SINGLE PRECISION, there is considerable decrease in solution time and the results are comparable with DOUBLE PRECISION solution.

b. As the problem size (i.e. NEQ * MBAND) increases with same MTOT, NEQB decreases and NBLOCK increases and corresponding solution time increases.

c. As MBAND decreases the NEQB increases reducing the solution time considerably.

CONCLUSIONS

The following conclusion can be drawn :

1. For both medium and large sized problems the block solver can be used.

2. For medium sized problems, profile solver takes lesser solution time as compared to that taken by block solver. However the total execution time taken by profile solver is much more as compared to that taken by block solver.

3. The large sized problems cannot be solved by profile solver. However these can be solved easily using block solver.

4. The size of the problem that can be solved, depends mainly on the out-of-core memory available with the system.

5. The block solver enlarges the usage of mini-computers in structural analysis. However, the solution time increases considerably.

6. For linear static problems, the solution time is about 50 to 70 percent of total execution time.

7. As the size of problem increases, the time of analysis also increases. If very small number of equations are accommodated in the block, the time increases exponentially.

8. For a given problem, the solution time decreases if more number of equations in a block can be accommodated or number of blocks are less.

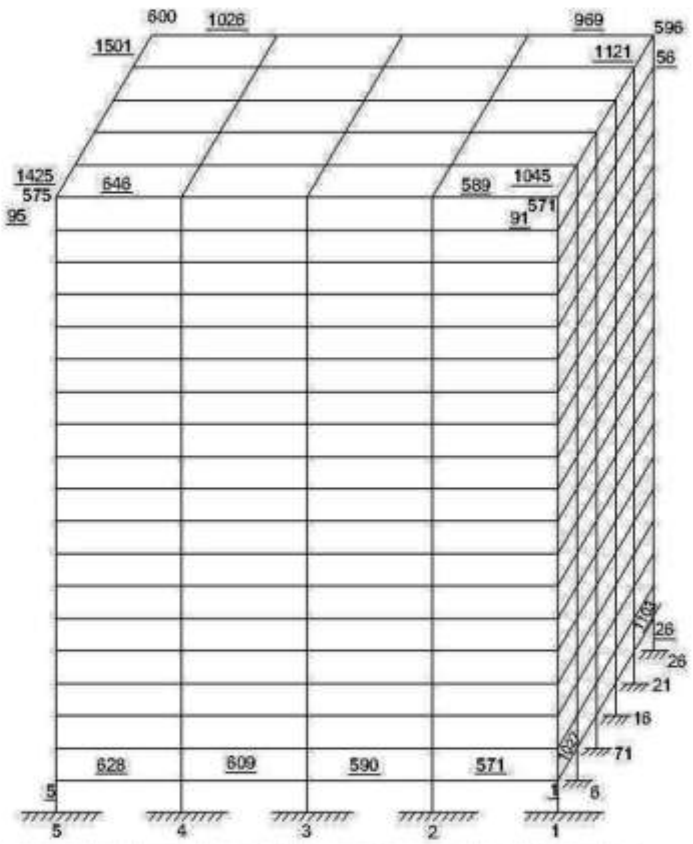


Fig. 5 A 19 Storey (5 Bay x 4 Bay) Frame for Test problem 2.

REFERENCES

[1]. ANSYS, "Engineering Analysis System - User's manual", Swanson Analysis System Inc., U.S.A., 1982.

[2]. BATHE, K.J., ADINA- A Finite Element Program for Automatic Dynamic Incremental Nonlinear Analysis. Report 92448-1, Acoustics and Vibration Laboratory, Department of Mechanical Engg., Massachusetts Institute of Technology, Cambridge, Mass.,1975.

[3]. BATHE, K.J., "Finite Element Procedures in Engineering Analysis", Prentice Hall, New Delhi, 1982.

[4]. BATHE, K.J., WILSON, E.L. and PETERSON, F.E., "SAP-IV, A Structural Analysis Program for Static and Dynamic Response of Linear System", Report No. EERC 73-11.Earthquake Engineering Research Center, University of California, Berkeley, 1974.

[5]HINTON, E. and OWEN, D.R.J., "Finite Element Programming", Academic Press, London, 1977.

[6]. HOOD, P., "A Frontal Solution Program for Unsymmetric Equations", International Journal of Numerical Methods in Engineering, 10, 379-399, 1976.

[7]. IRONS, B.M., "A Frontal Solution Program for Finite Element Analysis", International Journal for Numerical Methods in Engineering, 2, 5-32, Jan. 1970.

[8]. KRISHNAMOORTHY, C.S., "Finite Element Analysis", Tata McGraw Hill Publishing Company Ltd., New Delhi, 1987.

[9]. MEEK, J.L., "Matrix Structural Analysis", International Student Edition, McGraw Hill Kogakusha Ltd., Tokyo, 1971.

[10]. MEYER, C, "Special Problems Related to Linear Equation Solvers", ASCE Journal of the Structural Division, 101, No. ST4, 869-890, April 1975.

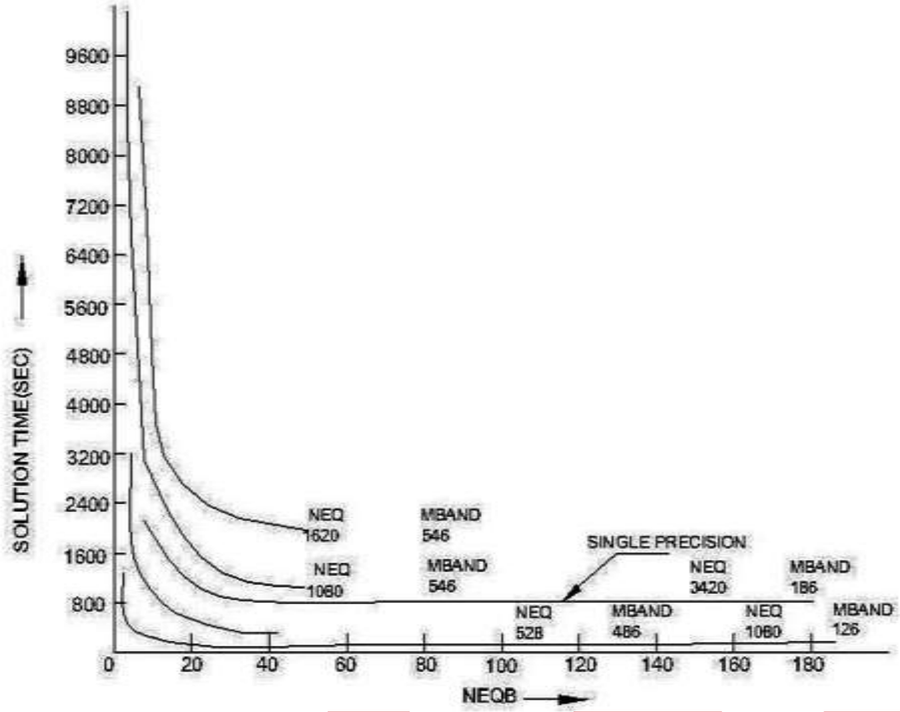
[11]. NATHAN, IDA and WILLIAM LORD, "Solution of Linear Equations for Small Computer Systems", International Journal for Numerical Methods in Engineering, 20, 625-641, 1984.

[12]. SLOAN, S.W., "A Fortran Program for Profile and Wavefront Reduction", International Journal for Numerical Methods in Engineering, 28, 2651-2679, 1989.

[13]. WILLIAM McGUIRE and RICHARD H. GALLACHER, "Matrix Structural Analysis", John Wiley and Sons, New York, 1979.

[14]. WILSON, E.L., BATHE, K.J. and DOHERTY, W.P., "Direct Solution of Large Systems of Linear Equations", Computers and Structures, 4, 363-372, 1974.

[15]. ZIENKIEWICZ, O.C., "Finite Element Method", Third Edition, Tata McGraw Hill Publishing Company Limited, New Delhi, Eighth Reprint, 1992.



Fig, 6 NEQB vs Solution Time (sec).

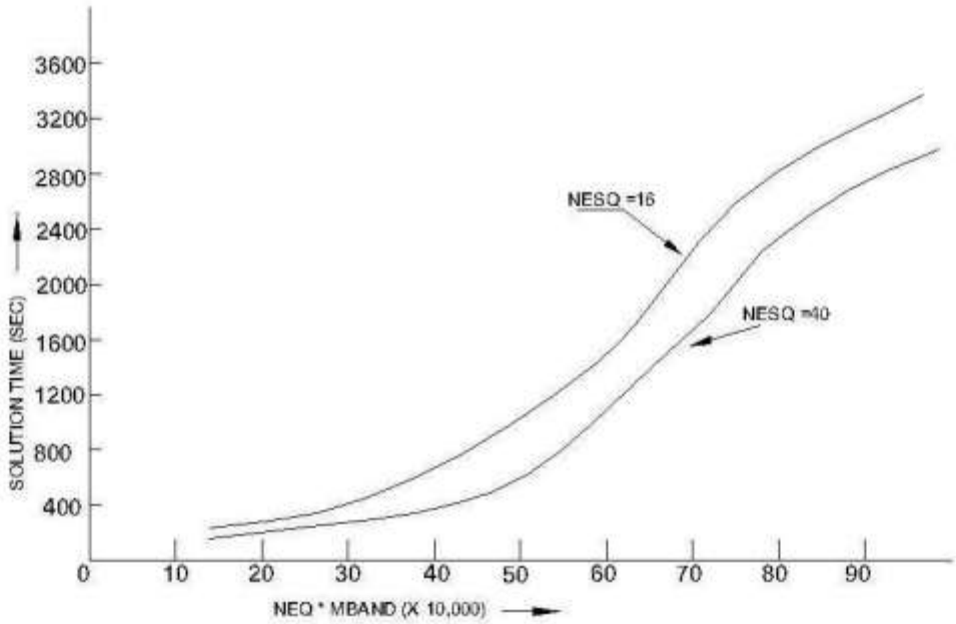


Fig. 7 NEQxMBANDvs Solution Time (sec)