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Odd Graceful Labeling Of Tensor Product Of Some Graphs

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ABSTRACT

A function *f* is called an odd-graceful labeling of a graph G=(V(G), E(G)) with p vertices and q edges, if there exists an injection function .. with each edge uv assigned the label $\varnothing : V(G) \rightarrow \{0, 1, 2, \ldots, 2q-1\}$ the resulting edge labels are {1, 3, 5,….., 2q-1}. The tensor product of two graphs, G and H, has a vertex set V(G)×V(H) and an edge between (u, v) and (u′, v′), iff both uu′ ∈ E(G) and vv′ ∈ E(H), here we denote tensor product by $\wedge$. In this paper, we prove $S_n \wedge S_m$ and $S_n \wedge P_m$ are odd graceful.

Keywords

Odd graceful labeling, Tensor product, Star Graph, Path.

Academic Discipline And Sub-Disciplines

Graph Theory

SUBJECT CLASSIFICATION

05C78

TYPE (METHOD/APPROACH)

Odd graceful labeling, Tensor Product

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INTRODUCTION

A graph G consist of a set V(G) of vertices and a set E(G) of edges, where $E(G) \subseteq V(G) \times V(G)$. When a single graph G is under consideration, we will write V = V(G) for it's vertex set and E = E(G) for it's edge set, furthermore, the order of G is defined as cardinality of V i.e |V| denoted by n often. If I a non negative integer f(u) is assigned to each vertex u, then the vertices are said to be labeled. G=(V,E) is itself a labeled graph if each edge e is given the value f(e)= |f(u)-f(v)|, where u and v are the end points of edge. Clearly every graph can be labeled in infinitely many ways. graph labeling is active research area in graph theory which has rigorous applications in coding theory, communication networks, optimal circuits layouts and graph decomposition problems. According to Beineke and Hegde [1] graph labeling serves as a frontier between number theory and the structure of graphs.

A function f is called an odd-graceful labeling of a graph G=(V, E) with p vertices and q edges, if there exists an injection function $\varnothing : V(G) \rightarrow \{0, 1, 2,\ldots, 2q-1\}$ with each edge uv assigned the label |f(u)-f(v)|, the resulting edge labels are {1, 3, 5,…., 2q-1}. A graph which consist odd graceful labeling is odd graceful graph.

In 1991, Gnanajothi [3] proved the graph $C_m \times K_2$ is odd graceful iff m even and the graph obtained from $P_n \times P_2$ by deleting an edge that joins to end points of the P_n paths, this graph is known as the ladder graph. Author proved that every graph with an odd cycle is not odd graceful. This labeling has been studied in several articles. In 2000, Kathiresan [4] used the notation $P_{n;m}$ to denote the graph obtained by identifying the end point of m paths each has length n. Chawathe and Krishna [5] have extended the definition of odd gracefulness the countably infinite graphs and showed that all countably infinite bipartite graphs which are connected and locally finite have odd graceful labeling.

The **path** $P_n(n\geq 2)$ has vertices $v_1, v_2,\ldots, v_n$ and edges $v_1v_2, v_2v_3,\ldots, v_{n-1}v_n$. The cycle on n-vertices $C_n(n\geq 3)$ consists of P_n plus an additional edge v_1v_n; cycles are odd or even, according as the number of vertices is odd or even.

The n-dimensional **star graph** is denoted by S_n. The vertex set V of S_n is $\{a_1,\ldots,a_n \mid a_1,\ldots,a_n$ is a permutation of 1, 2,...., n $\}$ and the edge set E is $\{(a_1a_2,\ldots, a_{i-1}a_ia_{i+1},\ldots, a_n, a_ia_2,\ldots, a_{i-1}a_1a_{i+1},\ldots, a_n) \mid a_1,\ldots, a_n \in V$ and $2 \leq i \leq n\}$. Clearly by definition, S_n contains n! vertices and each vertex is of degree (n-1).

The tensor product of two graphs G and H has a vertex set V(G) × V(H) and an edge between (u,v) and (u',v'), iff both $uu' \in E(G)$ and $vv' \in E(H)$, here we denote tensor product by $\wedge$.

In this paper, we prove $S_n \wedge S_m$ and $S_n \wedge P_m$ are odd graceful.

THE MAIN RESULTS

Theorem 1. The tensor product of S_n and S_m i.e $S_n \wedge S_m$ is odd graceful.

Proof. Let $G= S_n \wedge S_m$ be a graph with p vertices and q edges. The graph G is obtained by tensor product of S_n and S_m where n=1, 2,…., n and m=1, 2,…., m. S_n has vertex set $\{u_1, u_2,\ldots,u_n\}$ and S_m has $\{v_1, v_2,\ldots,v_n\}$. The graph G has q=2(n-1)(m-1) number of edges and p=n.m number of vertices, as shown in figure:

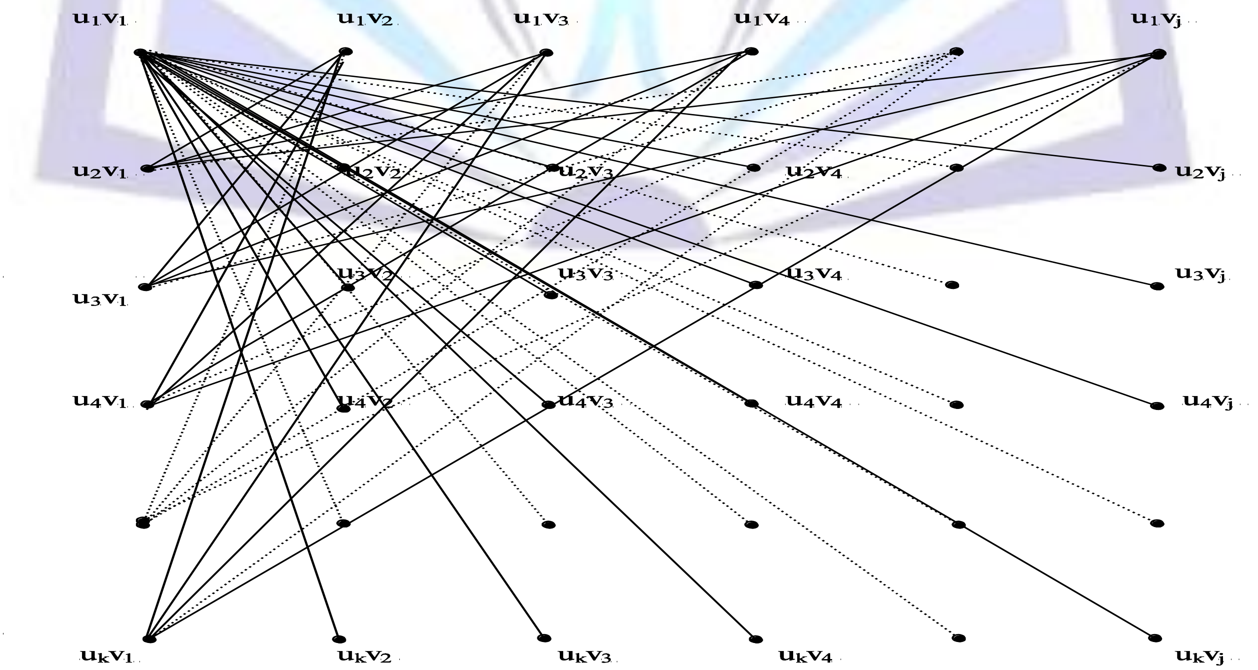

Fig 1: $S_n \wedge S_m$

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Define $\varnothing$: $V(G)\rightarrow\{0, 1, 2,\ldots, (2q-1)\}$ as following:

$\varnothing(u_1v_1)=2$

$\varnothing(u_kv_1)=(2q-1)-2(k-2)$ where, $2 \le k \le n$

$\varnothing(u_1v_2)=0$

$\varnothing(u_kv_j)=6(k-1)-3+2(j-2)+2(k-2)(m-4)$ where, $2 \le k \le n$, $2 \le j\le m$

$\varnothing(u_1v_j)= 2(n-1)(j-2)$, $2 \le j \le m$

In accordance with the above labeling pattern the graph under consideration admits odd graceful labeling.

Example :

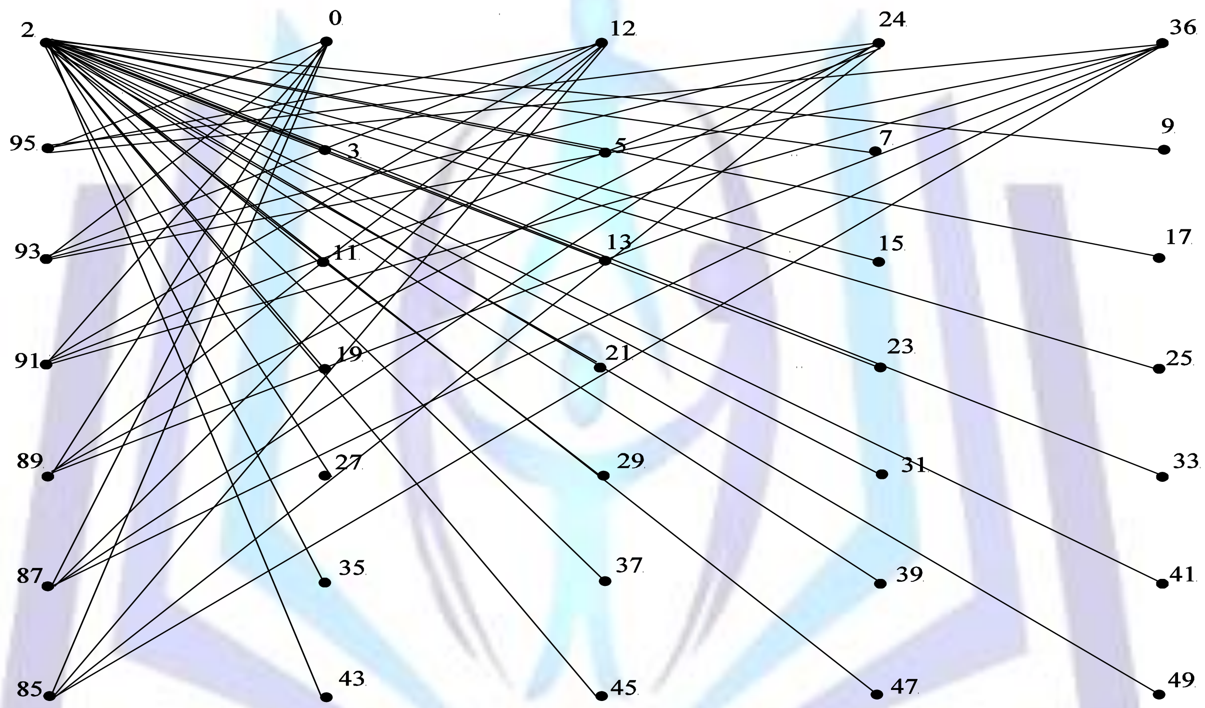

Fig 2: $S_7 \wedge S_5$

Theorem 2. The tensor product of S_n and P_m i.e $S_n \wedge P_m$ is odd graceful.

Proof. Let $G= S_n \wedge P_m$ be a graph with p vertices and q edges. The graph G obtained by tensor product of S_n and P_m where n=1, 2,…., n and m=1, 2,…., m. S_n has vertex set $\{u_1, u_2,\ldots,u_n\}$ and P_m has vertex set $\{v_1, v_2,\ldots,v_n\}$. The graph G has $q=2(n-1)(m-1)$ number of edges and $p=n.m$ number of vertices, as shown in figure:

Define $\varnothing$: $V(G) \rightarrow\{0, 1, 2,\ldots, (2q-1)\}$ as following:

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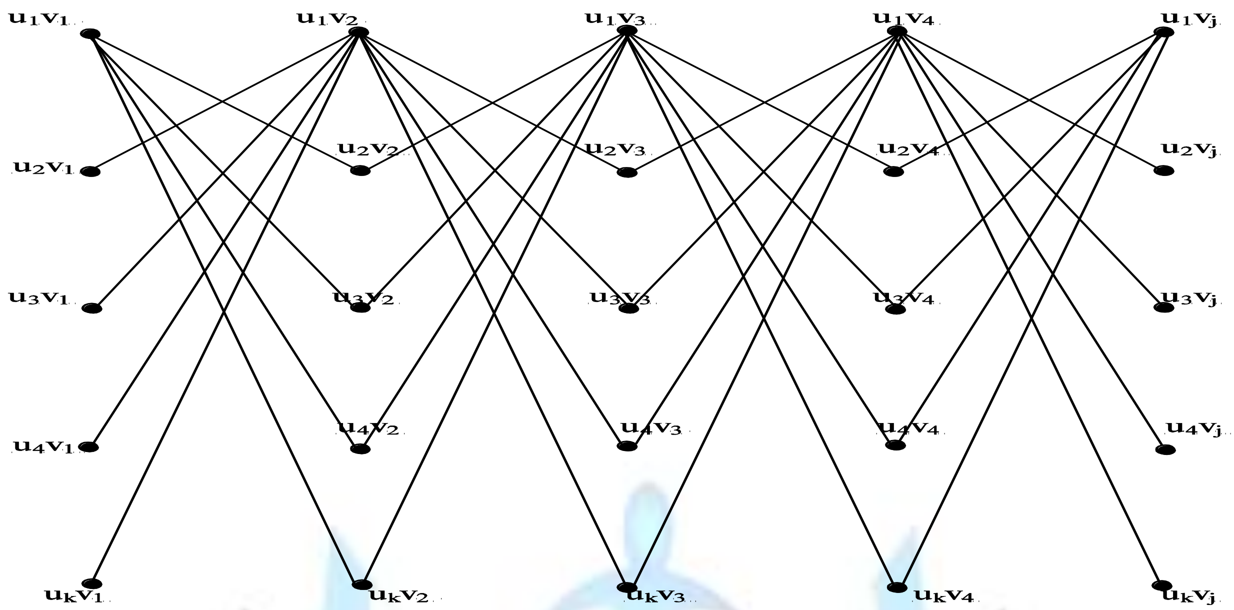

Fig 3: $S_n \wedge P_m$

$\varnothing(u_kv_1)=2(k-1)$ where, $1 \le k \le n$

$\varnothing(u_1v_2)=1$

$\varnothing(u_1v_{2j+1})=2(n-1)(j+1)$ where, $1 \le j \le m$

$\varnothing(u_1v_{2j+2})=2(n-1)(j+1)+1$, where, $1 \le j \le m$

$\varnothing(u_kv_2) = (2q-1)-2(k-1)$, where, $2 \le k \le n$

$\varnothing(u_kv_{2j+1})= (2q-1)-2(k-2)-2(n-1)-6(n-1)(j-1)+1$, where, $2\le k \le n$, $2 \le j \le m$

$\varnothing(u_k v_{2j})= (2q-1)-2(k-2)-2(n-1)-6(n-1)(j-1)+1-(2n-1)$**,** where, $2 \le k \le n$, $2 \le j \le m$

In accordance with the above labeling pattern the graph under consideration admits odd graceful labeling.

Example :

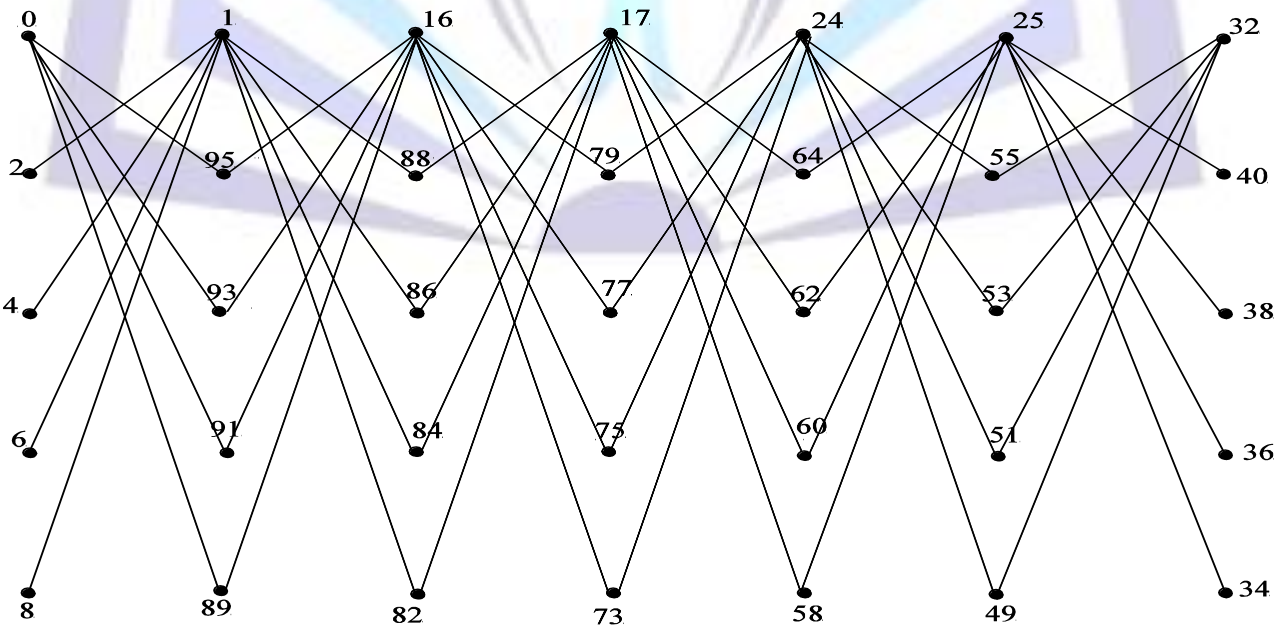

Fig 4: $S_5 \wedge P_7$

CONCLUSION

We have given a systematic approach to find odd graceful labeling of tensor products $S_n \wedge S_m$ and $S_n \wedge P_m$ and so these are odd graceful graphs.

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REFERENCES

[1] Beineke, L. W. and Hegde, S. M. 2001. Strongly Multiplicative Graphs, Discuss. Math. Graph Theory , 2(2001)-63-75.

[2] Chawathe, P. D. and Krishna, V., Odd Graceful Labelings of Countably Infinite Locally Finite Bipartite Graphs, Con--ference on Graph Theory and its Applications, March 2001, School of Mathematics, Anna University, Chennai.

[3] Gallian, J.A., A Dynamic Survey of Graph Labeling, The Electronic Journal of Combinotrics, 18(2011) DS6.

[4] Gnanajothi, R.B. 1991. Topics in Graph Theory, Ph.D Thesis, Madurai Kamaraj University, Madurai.

[5] Kathiresan, K. 2000. Two Classes of Graceful Graphs, Ars. Combin.55, 129-132.

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