ISSN 2277-3061

Hydromagnetic Stability of Streaming Compressible Fluid Cylinder Pervaded by Magnetic Field

Shimaa L. Azwz
Basic Science Department, Modern Academy for Engineering and Technology Cairo, Egypt
shimaa_1234@hotmail.com

Abstract

The Stability of MHD compressible streaming fluid cylinder of radius R_o endowed with surface tension and pervaded by axial magnetic field has been developed. The stability criterion is established in general form. The model is capillary unstable only in the axisymmetric mode *m*=0, the electromagnetic forces acting interior and exterior the fluid cylinder are stabilizing and the MHD stability is destabilizing for small wave length. In the latter case the instability shrinks with increasing the magnetic intensity. However the compressibility has a stabilizing tendency.

Keywords

Magneto hydrodynamic-compressible- streaming.

Academic Discipline and Sub-Disciplines

Applied Mathematics

SUBJECT CLASSIFICATION

Stability of liquid cylinder

TYPE (METHOD/APPROACH)

Application of stability of liquid cylinder

Council for Innovative Research
Peer Review Research Publishing System
Journal: INTERNATIONAL JOURNAL OF COMPUTERS & TECHNOLOGY
Vol 12, No.4
editor@cirworld.com
www.cirworld.com, member.cirworld.com

1. Introduction

The stability of a liquid column has been studied by Plateau and Savart (1873). Rayliegh (1945) derived the dispersion relation. Donnelly and Glaberson (1966) p(542) examined the type of perturbation on the boundary of the capillary instability of liquid jet. The analytical studies have been performed by Rayleigh (1945).

Such works have been exended by Chandrasekhar (1981) see also Radwan and Elazab (1990-2008) and Azwz (2008), (2010) and (2011).The present work is totally different those studied before since the velocity here is not solenoidal any more (i.e. $\nabla.\underline{u} \neq 0$) and also that the density of the fluid is not uniform.

2. Formulation of the problem

We consider a streaming (velocity $\underline{u}_o$) liquid cylinder of density ρ (of radius R_o) endowed with surface tension and pervaded by axial magnetic field for all modes of perturbation. The fluid is assumed to be compressible, inviscid and perfectly conducting. The fluid of density ρ is pervaded by the magnetic field.

$$\underline{H}_0 = (0,0,H_0) \qquad (1)$$

The surrounding tenoues medium around the cylinder is assumed to be pervaded by the magnetic field

$$\underline{H}_0^{vac} = (0,0,\alpha H_0) \qquad (2)$$

Under the present circumstances, the fundamental equations are given as follows.

$$\rho(\frac{\partial \underline{u}}{\partial t} + (\underline{u}.\nabla)\underline{u}) = -\nabla p + \mu(\nabla \wedge \underline{H}) \wedge \underline{H} \qquad (3)$$

$$\frac{\partial \rho}{\partial t} + \nabla.(\rho \underline{u}) = 0 \qquad (4)$$

$$p = C\rho^{\Gamma} \qquad (5)$$

$$\nabla.\underline{H} = 0 \qquad (6)$$

$$\frac{\partial \underline{H}}{\partial t} = \nabla \wedge (\underline{u} \wedge \underline{H}) \qquad (7)$$

In the vacuum region, the basic equations are

$$\nabla.\underline{H}^{vac} = 0 \qquad (8)$$

$$\nabla \wedge \underline{H}^{vac} = 0 \qquad (9)$$

$$p_s = T(\nabla.\underline{n}_s) \qquad (10)$$

$$\underline{n}_s = \frac{\nabla F(r,\varphi,z;t)}{|\nabla F(r,\varphi,z;t)|} \qquad (11)$$

$$F(r,\varphi,z;t) = 0 \qquad (12)$$

$$\underline{u}_o = (0,0,U) \qquad (13)$$

Here $\underline{u}$ and p are the fluid velocity vector and kinetic pressure, $\underline{H}$ is the magnetic field intensity. Equation (3) is the magnetodynamic vector equation of motion including the magnetodynamic (Lorentz) force $\mu(\nabla \wedge \underline{H}) \wedge \underline{H}$ and the gradient pressure force ∇p. Equation (4) is the continuity equation for compressible fluid. Equation (5) is the state equation. Equation (6) and (7) are the equations of the magnetic field in the liquid region. Equation (8) and (9) are the equations of vaccum region. Equation (10) gives the pressure due to the capillary force. Equation (11) is the unit normal vector $\underline{N}$ to the fluid - liquid interface and equation (12) is the equation of the boundary surface.

3. Unperturbed State

The unperturbed stationary state is studied upon considering the basic equations system (1) - (7). Equation (1) reduces to

T o identify this constant of integration we have to apply the balance of the pressure across the boundary surface at $r = R_o$. Taking into account equation (10) in the initial state gives.

$$\nabla \Pi_o = 0, \qquad (13)$$

$$\Pi_o = p_o + \frac{\mu}{2}(\underline{H}_o.\underline{H}_o) \qquad (14)$$

from which

$$p_o + \frac{\mu}{2} H_o^2 = const. \qquad (15)$$

$$p_{os} = \frac{T}{R_o} \qquad (16)$$

Consequently, the unperturbed pressure distribution is finally given by

$$p_o = \frac{T}{R_o} + \frac{\mu H_o^2}{2}(\alpha^2 - 1) \qquad (17)$$

Where the first term in the right side of equation (17) represent the capillary force contribution while the second term is due to the electromagnetic force influence inside and outside the fluid cylinder. In the unperturbed state the pressure p_o must be positive and so using equation (17), we find

$$\alpha \geq \sqrt{1 - \frac{2T}{\mu H_o^2 R_o}} \qquad (18)$$

4. Perturbation analysis

Due to a small perturbation of the initial state, every variable quantity $Q(r,\varphi,z,t)$ may be expressed in the axisymmetric mode, as

$$Q(r,\varphi,z,t) = Q_o(r) + \varepsilon(t) Q_1(r,\varphi,z) + ... \qquad (19)$$

Where the subscript o as usual characterizes the initial quantities while those with index unity are their increments. By substituting the expansion given in equation (19) into equations (3)-(10), the relevant perturbation equations in the fluid jet are

By an appeal to the expansion (2.13) for (2.1) - (2.7), the linearized perturbation equations are being

$$\frac{\partial \underline{u}_1}{\partial t} + (\underline{u}_o.\nabla)\underline{u}_1 - \frac{\mu}{\rho}(\underline{H}_o.\nabla)\underline{H}_1 = -\nabla \Pi_1 \qquad (20)$$

$$\frac{\partial \rho_1}{\partial t} + \nabla.(\rho_o \underline{u}_1) = 0 \qquad (21)$$

Where

$$\Pi_1 = \frac{p_1}{\rho} + \frac{\mu}{2\rho}(\underline{H}.\underline{H})_1$$

$$p_1 = a^2 \rho_1 \quad (22)$$

$$\frac{\partial \underline{H}_1}{\partial t} = (\underline{H}_o.\nabla)\underline{u}_1 - (\underline{u}_o.\nabla)\underline{H}_1 \quad (23)$$

$$\nabla.\underline{H}_1 = 0 \quad (24)$$

While in the vacuum region

$$\nabla.\underline{H}_1^{vac} = 0 \quad (25)$$

$$\nabla \wedge \underline{H}_1^{vac} = 0 \quad (26)$$

And along the fluid vacuum interface given by

$$p_{1s} = -\frac{T}{R_o^2}(\zeta_1 + R_o^2 \frac{\partial^2 \zeta}{\partial z^2} + \frac{\partial^2 \zeta}{\partial \varphi^2}) \quad (27)$$

Where ζ is the elevation of the perturbed interface given by

$$\zeta = \varepsilon_o \exp(i(kz) + \sigma t) \quad (28)$$

Where σ is the growth rate of instability, *k* is the longitudinal wave number and a^2 is the speed of sound.

From the point of view of the space- time dependence, every small increment $Q_1(r,\varphi,z;t)$ could be expressed as:

$$Q_1(r,\varphi,z;t) = \varepsilon_o Q_1(r)\exp[\sigma t + i(kz)] \quad (29)$$

By the use of the expansion (29), the linearized equations (20)-(27) are simplified and solved. Under the present circumstances the non-singular solution is given by

$$\rho_o a^2 (\nabla.\underline{u}_1) = -\sigma p_1 \quad (30)$$

$$(\sigma + \frac{\Omega_A^2}{\sigma})\underline{u}_1 = -\nabla \Pi_1 + \frac{i\Omega_A^2}{ka^2\rho_o} p_1 \underline{e}_z \quad (31)$$

$$\sigma \underline{H}_1 = ikH_o \underline{u}_1 + \frac{\sigma H_o}{\rho_o a^2} p \underline{e}_z \quad (32)$$

$$\Omega_A^2 = \frac{\mu H_o^2 k^2}{\rho} \quad (33)$$

$$\Pi_1 = \frac{p_1}{\rho_o} + \frac{\mu}{\rho_o}(\underline{H}_o.\underline{H}_1) \quad (34)$$

$$H_{1z} = \frac{H_o}{\rho_o a^2 \sigma}(\sigma + \frac{k^2 a^2}{\sigma}) p_1 \quad (35)$$

$$\Pi_1 = \frac{p_1}{\rho_o}\zeta \quad \text{With} \quad \zeta = 1 + \frac{\mu H_o^2}{\rho_o a^2 \sigma}\left[\sigma + \frac{k^2 a^2}{\sigma}\right] \quad (36)$$

ISSN 2277-3061

Substituting from equation (31) into (30), we get

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \Pi_1}{\partial r}\right)-\gamma^2\Pi_1=0 \quad (37)$$

With

$$\gamma^2=\eta^2+\beta^2 \quad (38)$$

$$\eta^2=k^2+\frac{\sigma^2}{a^2\zeta} \quad \text{and} \quad \beta^2=\frac{ikU}{a^2\sigma\zeta}(\sigma^2+\Omega_A^2) \quad (39)$$

By using the expansion (37), taking into account the dependence (26), the solution of (37) is given in terms of Bessel functions of first kind of order zero

$$\Pi_1=AI_o(\gamma r)\exp(ikz+\sigma t) \quad (40)$$

where $I_o(\gamma r)$ is the modified Bessel function of the first kind of order zero while *A* is an arbitrary constant to be determent The perturbed magnetic field in the region surrounding the fluid cylinder is obtained by solving the relevant perturbation equations (25) and (26). Equation (25) means that the perturbed magnetic field in the region surrounding the fluid can be derived from a scalar function Ψ_1, say

$$\underline{H}_1^{vac}=\nabla\Psi_1 \quad (41)$$

By combining equations (25) and (41), we get

$$\nabla^2\Psi_1=0 \quad (42)$$

Using the expansion (29) the solution of (41) is given in terms of Bessel functions

$$\Psi_1=BK_o(kr)\exp[ikz+\sigma t] \quad (43)$$

and consequently

$$\underline{H}_1^{vac}=B\nabla[K_o(kr)\exp[ikz+\sigma t] \quad (44)$$

Where $K_o(kr)$ is the modified Bessel function of the second kind of order zero and *B* is the unspecified constant. Finally,

$$p_{1s}=-\frac{T}{R_o^2}(1-k^2R_o^{\,2})\zeta \quad (45)$$

5. Boundary conditions

The solution of the above equations in the perturbed state must satisfy the following boundary conditions:

(1) The normal component of the velocity must be compatible with the velocity of the fluid particles across the boundary surface at $r=R_o$, i.e. $u_{1r}=\frac{dr}{dt}$.

$$A=\frac{-(\sigma^2+\Omega_A^2)}{\gamma I_o^{'}(\gamma r)}\varepsilon_o \quad (46)$$

(2) The normal component of the magnetic field $\underline{H}$ namely H_r must be continues across the perturbed boundary at $r=R_o$

$$B=\frac{i\varepsilon_o H_o\alpha}{K_o^{'}(x)} \quad (47)$$

ISSN 2277-3061

Therefore, the perturbed magnetic field external the fluid cylinder is given by

$$\underline{H}_1^{vac} = \frac{i\varepsilon_o H_o \alpha}{K_o^{'}(x)} \nabla[K_o(kr)\exp(\sigma t + ikz)] \qquad (48)$$

(3) The normal component of the stresses due to the kinetic pressure of the fluid and the magnetic pressure of the electromagnetic forces acting inside and outside the fluid column must be discontinues by the curvature pressure. This may be written as

$$p_1 + < \frac{\mu}{2}(\underline{H}.\underline{H})_1 >= p_{1s} \qquad (49)$$

After lengthy calculations, we finally arrive to

$$(\sigma + ikU)^2 = -\frac{T}{\rho_o R_o^3}(1-x^2)\frac{yI_o^{'}(y)}{I_o(y)} + \frac{\mu H_o^2}{\rho_o R_o^2}\left(-x^2 + \alpha^2 xy \frac{I_o^{'}(y)K_o(x)}{I_o(y)K_o^{'}(x)}\right) \qquad (50)$$

6. General discussions

Equation (50) is the capillary dispersion relation of fluid cylinder acting upon electromagnetic and capillary forces with uniform magnetic fields.

If we impose $H_o = 0,$ equation (50) reduces to

$$(\sigma + ikU)^2 = -\frac{T}{\rho_o R_o^3}(1-x^2)\frac{yI_o^{'}(y)}{I_o(y)} \qquad (51)$$

As the fluid is incompressible so a $\to \infty$ and in such case $y \approx x$, therefore equation (50) degenerates to

$$(\sigma + ikU)^2 = -\frac{T}{\rho_o R_o^3}(1-x^2)\frac{xI_o^{'}(x)}{I_o(x)} + \frac{\mu H_o^2}{\rho_o R_o^2}\left(-x^2 + \alpha^2 x^2 \frac{I_o^{'}(x)K_o(x)}{I_o(x)K_o^{'}(x)}\right) \qquad (52)$$

If we impose $H_o = 0,$ U=0 equation (52) reduces to that derived by Rayleigh (1945)

$$\sigma^2 = -\frac{T}{\rho_o R_o^3}(1-x^2)\frac{xI_1(x)}{I_o(x)} \qquad (53)$$

7. Hydrodynamic Instability

If the model under consideration is acting upon the capillary force only and other forces are neglected the dispersion relation for this case is given by (51). The discussions of this relation showed that the model is capillary stable in the symmetric mode *x*>1 and unstable if *x*<1 where *x*=1 is marginally stability state.

8. Magnetodynamic Stability

In the absence of the capillary force and the fluid cylinder only subjected to the electromagnetic forces interior and exterior the fluid, the dispersion relation

$$(\sigma + ikU)^2 = \frac{\mu H_o^2}{\rho_o R_o^2}\left(-x^2 + \alpha^2 xy \frac{I_o^{'}(y)K_o(x)}{I_o(y)K_o^{'}(x)}\right) \qquad (54)$$

The electromagnetic forces acting interior and exterior the fluid cylinder are stabilizing effect due to the fact that the applicable magnetic fields are axial and uniform.

Therefore, the model is purely stable under the influence of the electromagnetic force.

9. MHD Stability

In the case which the fluid cylinder is acting upon the capillary and electromagnetic forces , the model is unstable for small region while stable otherwise in the case of effect of capillary force while instability shrinks with increasing magnetic intensity and it may suppress the destabilizing character of the capillary force.

10. Numerical discussions

In order to verify the results obtained analytically concerning the acting different forces effects on the present model, it is found very important to discuss the dispersion relation (50) numerically. In order to do that we have to rewrite this dispersion relation in non-dimensional form, so we may insert such relation in the computer for making the numerical computation. Based on the input data, one has to check whether the values of $\sigma^2 / (T / \rho R_o^3)$ are positive or other wise. If the data are positive, then we have data for unstable regions. As values $\sigma^2 / (T / \rho R_o^3)$ are negative we put $\sigma = i\omega$ are the values of the oscillation frequency concerning the stable domains. In the transition from the negative values to positive values we have to path with the values of $\sigma = 0$ which means marginal stability states. The points at which the transition from stability regions to those of instability called the critical points.

The dispersion relation (50) has been formulated in the dimensionless form upon using the quantity $(T / \rho R_o^3)^{1/2}$ which has a unit of time as $\frac{1}{\sigma}$ and that the quantity $\sigma^2 / (\mu H_o^2 / \rho R_o^2)$ says H_G has a unit of the intensity of magnetic field. Consequently, equation (2.33) takes the form

$$\frac{(\sigma + ikU)^2}{(T / \rho R_o^3)} = -(1 - x^2)\frac{x L_{x,y}^o}{K_o^{'} L_y^o - s L_{x,y}^o} + \left(\frac{H_o}{H_G}\right)^2 \left(\frac{K_o L_{x,y}^o - L_y^o}{K_o^{'} L_y^o - s L_{x,y}^o}\right) \qquad (55)$$

The numerical calculations have been performed by inserting the dispersion relation (2.41) in the computer and computed. Taking into $\mathrm{K}_0'(x) = -\mathrm{K}_1(x)$. The calculations have been carried out for all short and long wave lengths as 0 ≤ x < 4.0 for several values of (H_o / H_G) = 0, 0.2 and 0.8.

ACKNOWLEDGMENTS

Our thanks to the experts who have contributed towards development of the template.

References

1. Chandrasekhar, S. and Fermi, E.,"Magnetic fields in spiral arms", Astrophys J., 116 (1953) 118-131.
2. Chandrasekhar, S., “Hydrodynamic and Hydromagnetic Stability” (Dover publ., New York, U. S. A) 1981.
3. Donelly, R. J and Glaberson, W.,Proc. Roy.Soc. London, 290A(1966)547.
4. Plateau, J., "Statique Expermintal et Theorique des Liquids Soumis aux Seules Molecularies", (Gauthier Villars, Paris, France)(1873).
5. Hasan A.A, Mekheimer Kh. S, and Azwz Sh. L." Hydromagnetic stability of self-gravitational oscillating streaming fluid jet pervaded by azimuthal varying magnetic field". International Journal of Mathematical Archive-2(4) 2011
6. Radwan, A. E. ., Elazab, S. S. and Demerdash A.A.,"Magneto-selfgravitational stability of triple superposed fluids" Phys. Scripta., (Sweden) 50 (1994) 142.
7. Radwan, A. E., Elazab, S. S. and Hydia,"Magnetohydrodynamics Stability of a Streaming annular cylindrical liquid surface", W. M., Phsica Scripta, 56 (1997) 193-199.
8. Radwan, A. E.,"Hydromagnetic stability of oscillating Hollow jet" Int. Engng. Sci., 36 (1998) 87.
9. Radwan, A. E. and Sileem, "Gravitational Instability of a Radiating Gas Cloud with variable streams", S.M., J. Chaos, Solitons and Fractals (London), 12 (2001) 1415-1429.
10. Radwan, A. E., Nuovo Cimento, 1178(2002)257.
11. Radwan, A. E. and Ogail,"Instability of a gas jet of zero inertia dispersed in a liquid using Lagragian's non linear differential equation", M. A., Nuovo Cimento, 118 B (2003) 713.
12. Radwan, A. E.,"Instability of a compressible annular magnetized fluid" J. Appl. Maths. Comp., 160 (2005) 283-301.
13. Radwan, A. E., Kyngpook Math. J. (Korea) 47 (2007) 455.
14. Radwan A.E. and Azwz Sh. L., " AJBAS 125-128, (2008)
15. Radwan, A. E. and Hasan, A.A., "Axisymmetric electrogravitational stability of fluid cylinder ambient with transverse varying oscillating field", (IAENG) 33(4) (2009) 2121-2132.
16. Rayleigh, J., "The Theory of Sound" (Dover Publ., New York, USA) (1945).