



## Topological Structure of Rough Soft Formal Context

FU Li

School of Mathematics and Statistics, Qinghai Nationalities University, China

### Abstract

In this paper, the topological structure is discussed in the rough soft formal context. The rough soft formal context is defined on the rough formal context with some soft operators, the topology and the topological space are given in the rough soft formal context. And some topological properties are discussed over the rough soft formal context.

**Keywords:** rough soft formal context, topological space, soft open(closed) set, soft interior point.



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## 1 Introduction

Formal concept context, rough set, soft set are extensively applied in the uncertainty reasoning, the uncertainty of data while modeling the problems in engineering, physics, computer sciences, economics, social sciences, medical science and many other diverse fields. These may be due to the uncertainties of natural environmental phenomena, such as vagueness or uncertainty in the boundary between states or between urban and rural areas and son on. More and more researchers study these uncertainties.

In 1982, R.Wille<sup>[1]</sup> proposed a new model to represent the formal concepts, he named it as formal concept analysis, which is a binary relation between a set of objects and a set of attributes. The formal concept analysis is based on mathematical order theory and based on the formal context  $(G, M, I)$ . The main goal is to reveal the hierarchical structure of the formal concepts and to investigate the dependencies among attributes. The family of all formal concepts is a complete lattice, which is an effective method for several real-world applications in data analysis, such as object-oriented databases, inheritance lattices, mining for association rules, generating frequent sets etc(see [2-11]). One of the important challenges in data handling is generating the concept lattice of the binary relation. The theory of rough sets, proposed by Z.Pawlak<sup>[6]</sup>, is an extension of set theory for the study of intelligent systems characterized by insufficient and incomplete information.

In 1999, Molodsov [12] initiated a novel concept called soft set theory, which is a completely new approach for modeling vagueness and uncertainty. The soft set theory has a rich potential for applications in several directions, few of which had been shown by Molodsov in [12]. After Molodsov's work, some different applications of soft sets were studied in [14, 15]. Furthermore, Maji, Biswas and Roy worked on soft set theory in [16]. Also Maji et al. presented the definition of fuzzy soft set in [17]. Roy et al. presented some applications of this notion to decision making problems in [18]. Recently, the many authors discuss the soft set, research on the soft set theory is progressing rapidly, for example, the concepts of soft semi-ring, soft group, soft BCK/BCI-algebra, soft BL-algebra, and fuzzy soft group etc. have been proposed and investigated (see [16-25] respectively). In[26], authors defined the rough formal context, and discussed its properties. In [27], Shabir and Naz initiated the study of soft topological spaces. In [28], authors continued investigating the properties of soft topological spaces. In[29], author defined the rough soft formal context, and discuss the rough properties of rough formal context in soft set.

In this paper, we discuss the topological structure of the rough soft formal context. The rest of this paper is organized as following. in section 2, we review some basic concepts and properties of rough concept formal and soft sets. In section 3, we define the topological structure over the rough soft formal context and discuss its topological properties. Conclusions are given in section 4.

## Basic knowledge

**Definition 2.1**<sup>[26]</sup> Let  $(G, M, R)$  be an information system, where  $G = \{a_1, \dots, a_m\}$  is an object set,  $M = \{x_1, \dots, x_n\}$  is an attribute (property) set,  $R$  is an equivalent relation on  $G$ ,  $\forall A \subseteq G$ , we can define the upper and the lower approximation of  $A$  about  $R$ :

$$\overline{A}_R = \bigcup \{Y \in G/R \mid Y \cap A \neq \emptyset\} = \{x \in G \mid [x]_R \cap A \neq \emptyset\};$$

$$\underline{A}_R = \bigcup \{Y \in G/R \mid Y \subseteq A\} = \{x \in G \mid [x]_R \subseteq A\}.$$

$\overline{A}_R, \underline{A}_R$  are called  $R$ - upper approximation and  $R$ - lower approximation of  $A$ .  $\overline{A}, \underline{A}$  are denoted simply respectively. If  $\overline{A} = \underline{A}$ , we say that  $A$  is definable, otherwise,  $A$  is rough.

Similarly, we can define the upper and lower approximation of attributes set  $B \subseteq M$  about an equivalent relation on  $M$ .

**Definition 2.2**<sup>[12]</sup> Let  $U$  be an initial universe set and  $E$  be a set of parameters. Let  $\mathbf{P}(U)$  denotes the power set of  $U$  and  $A \subseteq E$ . Then a pair  $(F, A)$  is called a **soft set** over  $U$ , where  $F: A \rightarrow \mathbf{P}(U)$  is a mapping.

That is, the soft set is a parameterized family of subsets of the set  $U$ . Every set  $F(e), \forall e \in E$ , from this family may be considered as the set of  $e$ -elements of the soft set  $(F, E)$ , or considered as the set of  $e$ -approximate elements of the soft set. According to this manner, we can view a soft set  $(F, E)$  as consisting of collection of approximations:  $(F, E) = \{F(e) \mid e \in E\} = \{(F(e), e) \mid e \in E\}$ .

**Definition 2.3**<sup>[29]</sup> Let  $(G, M, R)$  is a rough formal context,  $G$  is objects set, is also called the universe,  $M$  is attributes set. A pair  $(F, B)$  is a soft set over  $G$ , where  $B \subseteq M$ , and  $F: B \rightarrow \mathbf{P}(G)$  is a set-value mapping over



$G$ , furthermore, the lower and upper rough approximations of pair  $(F, B)$  are denoted by  $\underline{R}(F, B) = (\underline{F}, B)$ ,  $\overline{R}(F, B) = (\overline{F}, B)$ , which are soft sets over  $G$  with the set-valued mappings given by  $\underline{F}(x) = \underline{B}(F(x))$  and  $\overline{F}(x) = \overline{B}(F(x))$ , where  $x \in B$ . The operators  $\underline{R}, \overline{R}$  are called the lower and upper rough approximation operators on soft set  $(F, B)$ .

If  $\overline{R} = \underline{R}$ , we say that the soft set  $(F, B)$  is definable, otherwise,  $(F, B)$  is rough.

we call such quadruple tuple  $(G, M, R, F)$  as **rough soft formal context**, and, such soft set  $(F, B)$  on the rough soft formal context  $(G, M, R, F)$  which is called **rough soft formal set**.

Obviously,  $\forall x \in B \subseteq M, F(x) \subseteq G$  is a parameterized family of subsets of  $G$ , and  $F(x)$  is the set of  $x$ -approximate elements in  $(G, M, R, F)$ .

**Example 1** Let  $(G, M, R, F)$  be a rough soft formal context, where  $G = \{h_1, h_2, h_3, h_4, h_5\}, M = \{e_1, \dots, e_5\}$ , in which  $e_1$  stands for "expensive",  $e_2$ : "beautiful",  $e_3$ : "wooden",  $e_4$ : "cheap",  $e_5$ : "in the green surrounding".

Let  $F_1, F_2 : M \rightarrow \mathcal{P}(G)$ , and suppose that:

$$F_1(e_1) = \{h_2, h_4\}, F_1(e_2) = \{h_1, h_3\}, F_1(e_3) = \emptyset, F_1(e_4) = \{h_1, h_3, h_5\}, F_1(e_5) = \{h_1\}.$$

**Definition 2.4**<sup>[29]</sup> Let  $(G, M, R, F)$  be a rough soft formal context over the objects set  $G$ , and attributes set  $M$ .  $B_1, B_2 \subseteq M$ ,  $(F, B_1)$  and  $(G, B_2)$  are two soft sets over  $G$  on the rough soft formal context  $(G, M, R, F)$ .  $F_1 : B \rightarrow \mathcal{P}(G)$  is a set-value mapping over  $G$  on  $(G, M, R, F)$ . (i) If  $B_1 \subseteq B_2$ , and  $F(x) \subseteq F_1(x), \forall x \in B_1 \subseteq B_2$ , then the soft sets  $(F, B_1)$  is a **soft subset** of the soft set  $(F_1, B_2)$ , denoted as  $(F, B_1) \tilde{\subset} (F_1, B_2)$ .

(ii) Two soft sets  $(F, B_1)$  and  $(F_1, B_2)$  on the rough soft formal context  $(G, M, R, F)$  are said **soft equal**, if  $(F, B_1) \tilde{\subset} (F_1, B_2)$ , and  $(F_1, B_2) \tilde{\subset} (F, B_1)$ . We simply denote by  $(F, B_1) = (F_1, B_2)$ .

**Definition 2.5**<sup>[29]</sup> Let  $(F, B)$  is a rough soft formal set over the rough soft formal context  $(G, M, R, F)$ ,

(1) The **relative complement** of  $(F, B)$  is denoted by  $(F, B)^c$  and is defined by  $(F, B)^c = (F^c, B)$ , where  $F^c : B \rightarrow \mathcal{P}(G)$  and  $F^c(x) = G - F(x), \forall x \in B$ .

Clearly,  $((F, B)^c)^c = (F, B)$ .

(2)  $(F, B)$  is said to be a **relative null rough soft formal set** denoted by  $\mathcal{N}$ , if  $\forall x \in B, F(x) = \emptyset$ , if  $B = M$ , then is called **absolute null rough soft formal set**.

(3)  $(F, B)$  is said to be a **relative whole rough soft formal set** denoted by  $\tilde{G}$ , if  $\forall x \in B, F(x) = G$ .

Consider example 1, let  $B = \{e_1, e_2\}$ , then  $F_1^c(e_1) = G - F_1(e_1) = \{h_1, h_3, h_5\}$ ,  $F_1^c(e_2) = G - F_1(e_2) = \{h_2, h_4, h_5\}$ , so,  $(F_1, B)^c = (F_1^c, B) = \{(\{h_1, h_3, h_5\}, e_1), (\{h_2, h_4, h_5\}, e_2)\}$ . If Let  $B_1 = \{e_1, e_3\}, B_2 = \{e_2, e_4, e_5\}$ , then  $(F_1, B_1) \subseteq (F_2, B_1)$ , and  $(F_1, B_2) = (F_2, B_1)$ .

**Definition 2.6**<sup>[29]</sup> Let  $(G, M, R, F)$  be the rough soft formal context,  $F_1, F_2 : B \rightarrow \mathcal{P}(G)$  are two set-value mapping over  $G$  on  $(G, M, R, F)$ .  $(F_1, B_1)$  and  $(F_2, B_2)$  are two rough soft formal sets over  $(G, M, R, F)$ .

(i) The **union** of  $(F_1, B_1)$  and  $(F_2, B_2)$  is the rough soft formal set  $(H, C)$ , where  $C = B_1 \cup B_2$ , and



$\forall e \in C$ , denoted as  $(F_1, B_1) \cup (F_2, B_2) = (H, C) = (H, B_1 \cup B_2)$ , where

$$H(e) = \begin{cases} F_1(e), & \text{if } e \in B_1 - B_2 \\ F_2(e), & \text{if } e \in B_2 - B_1 \\ F_1(e) \cup F_2(e), & \text{if } e \in B_1 \cap B_2 \end{cases}$$

(ii) The **intersection** of  $(F_1, B_1)$  and  $(F_2, B_2)$  is the soft rough formal set  $(H, C)$  is denoted as  $(F_1, B_1) \cap (F_2, B_2)$  and is defined as  $(F_1, B_1) \cap (F_2, B_2) = (H, C)$ , where  $C = B_1 \cap B_2$ , and  $\forall e \in C, H(e) = F_1(e) \cap F_2(e)$ .

### 3 The topology over the rough soft formal context

**Definition 3.1** Let  $\mathbb{T} = (G, M, R, F)$  be a rough soft formal context over the object set  $G$  and attributes set  $M$ ,  $B_i \subseteq M$ ,  $\tau = \{(F_i, B_i) \mid (F_i, B_i) \text{ is a soft set over } G\}$  which is the collection of soft sets on the rough soft formal context  $(G, M, R, F)$ , if

(1)  $(1) \ \emptyset, \tilde{G}$  belong to  $\tau$ .

(2) The union of any number of soft sets in  $\tau$  belongs to  $\tau$ , that is,  $\tau$  is closed for the any union of soft sets over  $\mathbb{T}$ .

(3) The intersection of any two soft sets in  $\tau$  belongs to  $\tau$ , that is,  $\tau$  is closed for the finite intersection of soft sets over  $\mathbb{T}$ .

Then The collection  $\tau$  is called a **soft topology** over the rough soft formal context  $\mathbb{T}$  (simply called **soft rough topology**). The triplet  $(G, \tau, M)$  is called a **soft topological space** over the rough soft formal context  $\mathbb{T}$  (simply called **soft rough topological space**), and the members of  $\tau$  are soft open sets in  $\mathbb{T}$ , the relative complement  $(F, B)^c = (F^c, B)$  is said to be a soft closed set in  $\mathbb{T}$  if  $(F, B)^c \in \tau$ .

**Proposition 3.1** Let  $\mathbb{T} = (G, M, R, F)$  be a rough soft formal context, and  $(G, \tau, M)$  be a soft rough topological space over the rough soft formal context  $\mathbb{T} = (G, M, R, F)$ , then

(1)  $\emptyset, \tilde{G}$  are closed soft sets over  $\mathbb{T}$ .

(2) The union of any two soft closed sets in  $\tau$  belongs to  $\tau$ , that is,  $\tau$  is closed for the finite union of soft closed sets over  $\mathbb{T}$ .

(3) The intersection of any number of soft closed sets in  $\tau$  belongs to  $\tau$ , that is,  $\tau$  is closed for the any intersection of soft closed sets over  $\mathbb{T}$ .

**Example 2** Let  $G$  be an initial universe set,  $M$  be a set of attributes, and  $\tau = \{\emptyset, \tilde{G}\}$  is a trivial soft rough topology over  $G$ ; if we denote  $\tau$  as the collection of all soft sets which determined by the power set of  $G$ , that is,  $\tau = \{(F_i, M) \mid F_i : M \rightarrow \wp(G)\}$ , then  $\tau$  is a discrete soft rough topology over  $\mathbb{T}$ .

**Example 3** Let  $(G, \tau, M)$  be a soft rough topological space over rough soft formal context  $\mathbb{T}$ , in which  $G = \{h_1, h_2, h_3\}$  is an initial universe set,  $M = \{e_1, e_2\}$  is a set of attributes, then the collection

$\{\emptyset, \tilde{X}, (F_1, M_1), (F_2, M_2), (F_3, M), (F_4, M), (F_5, M), (F_6, M)\}$  is a soft rough topology on  $\mathbb{T}$ , in which the subsets  $M_1 = \{e_1\}, M_2 = \{e_2\}$ , and soft sets:



$$\begin{aligned} & \{e_1\}, M_2 = \{e_2\}, \text{ and soft sets:} \\ & (F_1, M_1) = \{(\{h_2\}, e_1)\}, (F_2, M_2) = \{(\{h_1\}, e_2)\}, \\ & (F_3, M) = \{(\{h_1, h_2\}, e_1), (G, e_2)\}, \\ & (F_4, M) = \{(\{h_1, h_2\}, e_1), (\{h_1, h_3\}, e_2)\}, \\ & (F_5, M) = \{(\{h_2\}, e_1), (\{h_1, h_2\}, e_2)\}, \\ & (F_6, M) = \{(\{h_2\}, e_1), (\{h_1\}, e_2)\}. \end{aligned}$$

**Note:** We can understand  $(F_1, M_1) = \{(\{h_2\}, e_1)\}$  as  $(F_1, M_1) = \{(\{h_2\}, e_1), (\emptyset, e_2)\}$ , so, for simply, in the following, we do not take the subset of the attributes set  $M$ .

**Example 4** Let  $(G, \tau, M)$  be a soft rough topological space over rough soft formal context  $T = (G, M, R, F)$  which  $G$  is an initial universe set,  $M$  is a set of attributes, then the collection  $\tau_\alpha = \{F(\alpha) \mid (F, B) \in \tau, \alpha \in B \subseteq M\}$  for each  $\alpha \in B$  is a soft rough topology on  $T$ .

Obviously, because  $\tau$  is a soft rough topology over  $T$ ,

- (i) by  $\emptyset, \tilde{G} \in \tau$ , we know  $\emptyset_\alpha, \tilde{G}_\alpha \in \tau_\alpha$ ;
  - (ii) By  $(F_1, B_1), (F_2, B_2) \in \tau$  can infer  $F_1(\alpha), F_2(\alpha) \in \tau_\alpha$ , and  $(F_1, B_1) \cap (F_2, B_2) \in \tau$  can infer  $F_1(\alpha) \cap F_2(\alpha) \in \tau_\alpha$ ;
  - (iii) By  $(F_i, B_i) \in \tau$  can infer  $F_i \in \tau_\alpha$ , and  $(F_i, B_i) \in \tau$  can infer  $\bigcup_i F_i(\alpha) \in \tau_\alpha$ .
- Consider example 3,  $\tau_{e_1} = \{\emptyset, \tilde{M}, \{h_2\}, \{h_1, h_2\}\}$ ,  $\tau_{e_2} = \{\emptyset, \tilde{M}, \{h_1\}, \{h_1, h_2\}, \{h_1, h_3\}\}$ .

**Proposition 3.2** Let  $(G, \tau_1, M), (G, \tau_2, M)$  be two soft rough topological spaces over the rough soft formal context  $T = (G, M, R, F)$ , then  $(G, \tau_1 \cap \tau_2, M)$  is a soft topological space over  $T$ .

**Proof** (i) Clearly,  $\emptyset, \tilde{G} \in \tau_1 \cap \tau_2$ ;

(ii) Let  $(F_1, B_1), (F_2, B_2) \in \tau_1 \cap \tau_2$ , then  $(F_1, B_1), (F_2, B_2) \in \tau_1$ , and  $(F_1, B_1), (F_2, B_2) \in \tau_2$  since  $(F_1, B_1) \cap (F_2, B_2) \in \tau_1$  and  $(F_1, B_1) \cap (F_2, B_2) \in \tau_2$ ,  $(F_1, B_1) \cap (F_2, B_2) \in \tau_1 \cap \tau_2$ ;

(iii) Suppose that  $\{(F_i, B_i) \mid i \in I\}$  be a family of soft set in  $\tau_1 \cap \tau_2$ , then  $(F_i, B_i) \in \tau_1$ , and  $(F_i, B_i) \in \tau_2$ , for all  $i \in I$ , so  $\bigcup_{i \in I} (F_i, B_i) \in \tau_1$ , and  $\bigcup_{i \in I} (F_i, B_i) \in \tau_2$ , Hence  $\bigcup_{i \in I} (F_i, B_i) \in \tau_1 \cap \tau_2$ .



**Remark** Let  $(G, \tau_1, M), (G, \tau_2, M)$  be two soft rough topological spaces over the rough soft formal context  $\mathcal{T} = (\mathcal{G}, \mathcal{M}, \mathcal{R}, \mathcal{F})$ , however,  $(G, \tau_1 \cup \tau_2, M)$  can be not a soft rough topological space over  $\mathcal{T}$ .

When  $(F_1, B_1), (F_2, B_2) \in \tau_1 \cup \tau_2$ , and  $(F_1, B_1), (F_2, B_2) \in \tau_1 - \tau_2$  or  $(F_1, B_1), (F_2, B_2) \in \tau_2 - \tau_1$ , or  $(F_1, B_1), (F_2, B_2) \in \tau_1 \cap \tau_2 \subseteq \tau_1 \cup \tau_2$ , then  $(G, \tau_1 \cup \tau_2, M)$  must be a soft rough topology over  $\mathcal{T}$ , but if  $(F_1, B_1) \in \tau_1$ , and  $(F_2, B_2) \in \tau_2$ , then  $(G, \tau_1 \cup \tau_2, M)$  can be not a soft rough topology over  $\mathcal{T}$ . For example:

In example 3, let  $\tau_1 = \{\emptyset, \tilde{X}, (F'_1, M_1), (F'_2, M_2), (F'_3, M)\}$  is a soft rough topology on  $\mathcal{T}$ , in which  $(F'_1, M_1) = \{(\{h_1\}, e_1)\}, (F'_2, M_2) = \{(\{h_2, h_3\}, e_2)\}, (F'_3, M) = \{(\{h_1\}, e_1), (\{h_2, h_3\}, e_2)\}$ . Then  $\tau \cup \tau_1 = \{\emptyset, \tilde{X}, (F_1, M_1), (F_2, M_2), (F_3, M), (F_4, M), (F_5, M), (F_6, M), (F'_1, M_1), (F'_2, M_2), (F'_3, M)\}$ , consider  $(F_4, M) \cap (F'_3, M) = \{(\{h_1\}, e_1), (\{h_3\}, e_2)\} \notin \tau \cup \tau_1$ , so  $\tau \cup \tau_1$  is not a soft rough topology on  $\mathcal{T}$ .

**Definition 3.2** Let  $\mathbb{T} = (G, M, R, F)$  be a rough soft formal context, and  $(G, \tau, M)$  be a soft topological space over  $\mathbb{T}$  which  $G$  is an initial universe set,  $M$  is a set of attributes, and  $(F, B)$  is a soft set over  $M$ . The **soft closure** of  $(F, B)$ , denoted by  $\overline{(F, B)}$  is the intersection of all soft closed super set of soft set  $(F, B)$ , that is  $\overline{(F, B)} = \bigcap \{(F_i, B_i) \mid (F, B) \subseteq (F_i, B_i) \in \tau\}$ . Clearly,  $\overline{(F, B)}$  is the smallest soft closed set over  $\mathbb{T}$  which contain the soft set  $(F, B)$ .

Consider example 3,  $\overline{(F_6, M)} = \{(\{h_2\}, e_1), (\{h_1\}, e_2)\}$ .

**Theorem 3.1** Let  $(G, \tau, M)$  be a soft rough topological space over  $\mathbb{T}$  which  $\mathbb{T} = (G, M, R, F)$  is a rough soft formal context,  $G$  is an initial universe set,  $M$  is a set of attributes, and  $(F_i, B_i), B_i \subseteq M, i = 1, 2$  is a soft set over  $G$ . Then

- (i)  $\overline{\emptyset} = \emptyset, \overline{\tilde{G}} = \tilde{G}$ ;
- (ii)  $(F, B) \tilde{\subseteq} \overline{(F, B)}$ ;
- (iii)  $\overline{(F, B)}$  is a soft closed set over  $\mathcal{T}$  if and only if  $(F, B) = \overline{(F, B)}$ ;
- (iv)  $\overline{\overline{(F, B)}} = \overline{(F, B)}$ ;
- (v)  $(F_1, B_1) \tilde{\subseteq} (F_2, B_2)$  implies  $\overline{(F_1, B_1)} \tilde{\subseteq} \overline{(F_2, B_2)}$ ;
- (vi)  $\overline{(F_1, B_1) \cup (F_2, B_2)} = \overline{(F_1, B_1)} \cup \overline{(F_2, B_2)}$ ;
- (vii)  $\overline{(F_1, B_1) \cap (F_2, B_2)} \tilde{\subseteq} \overline{(F_1, B_1)} \cap \overline{(F_2, B_2)}$ .



**Proof** By definition of soft closure of  $(F, B)$ ,  $\overline{(F, B)} = \bigcap_{(F_i, B_i) \in \tau} \{(F, B) \tilde{\subset} (F_i, B_i)\}$ ,  
 (i),(ii) are obvious.

(iii) If  $(F, B)$  is a soft closed set over  $\mathcal{T}$ , then  $(F, B)$  is itself a soft closed set over  $\mathcal{T}$  which contained  $(F, B)$ , so,  $(F, B)$  is the smallest soft closed containing  $(F, B)$  over  $\mathcal{T}$ , and  $(F, B) = \overline{(F, B)}$ ;

(iv)  $\overline{(F, B)}$  is a soft closed set, hence  $\overline{\overline{(F, B)}} = (F, B)$ .

(v) Suppose that  $(F_1, B_1) \tilde{\subset} (F_2, B_2)$ , then every closed super set of  $(F_2, B_2)$  is also contain  $(F_1, B_1)$ . Hence the intersection of soft closed super sets of  $(F_1, B_1)$  is contained in the soft intersection of soft closed super sets of  $(F_2, B_2)$ . Thus  $\overline{(F_1, B_1)} \tilde{\subset} \overline{(F_2, B_2)}$ .

(vi) Since  $(F_1, B_1) \tilde{\subset} (F_2, B_2) \cup (F_1, B_1)$ ,  $(F_2, B_2) \tilde{\subset} (F_1, B_1) \cup (F_2, B_2)$  implies  $\overline{(F_1, B_1)} \tilde{\subset} \overline{((F_1, B_1) \cup (F_2, B_2))}$ , and  $\overline{(F_2, B_2)} \tilde{\subset} \overline{((F_1, B_1) \cup (F_2, B_2))}$ . Conversely, since  $(F_1, B_1) \tilde{\subset} \overline{(F_1, B_1)}$ ,  $(F_2, B_2) \tilde{\subset} \overline{(F_2, B_2)}$ ,

so  $(F_1, B_1) \cup (F_2, B_2) \tilde{\subset} \overline{(F_1, B_1)} \cup \overline{(F_2, B_2)}$ , and  $\overline{(F_1, B_1)} \cup \overline{(F_2, B_2)}$  is soft closed set, then  $\overline{(F_1, B_1) \cup (F_2, B_2)} \tilde{\subset} \overline{(F_1, B_1)} \cup \overline{(F_2, B_2)}$ . Thus,  $\overline{(F_1, B_1) \cup (F_2, B_2)} = \overline{(F_1, B_1)} \cup \overline{(F_2, B_2)}$

(vii) Since  $((F_1, B_1) \cap (F_2, B_2)) \tilde{\subset} (F_1, B_1)$ , and  $((F_1, B_1) \cap (F_2, B_2)) \tilde{\subset} (F_2, B_2)$  implies  $\overline{((F_1, B_1) \cap (F_2, B_2))} \tilde{\subset} \overline{(F_1, B_1)}$ , and  $\overline{((F_1, B_1) \cap (F_2, B_2))} \tilde{\subset} \overline{(F_2, B_2)}$ . Therefore,  $\overline{(F_1, B_1) \cap (F_2, B_2)} \tilde{\subset} \overline{(F_1, B_1)} \cap \overline{(F_2, B_2)}$ .

**Definition 3.3** Let  $(G, \tau, M)$  be a soft rough topological space over  $\mathcal{T}$  which  $\mathcal{T} = (G, M, R, F)$  is a rough soft formal context,  $G$  is an initial universe set,  $M$  is a set of attributes, and  $(F, B)$  is a soft set over  $G$ ,  $x \in G$ , then **soft interior** of soft set  $(F, B)$  over  $G$  is denoted by  $(F, B)^\circ$  and is defined as the union of all soft open set contained in  $(F, B)$ , that is  $(F, B)^\circ = \bigcup \{(F_i, B_i) \mid (F_i, B_i) \tilde{\subset} (F, B) \in \tau\}$ . Such  $(F, B)^\circ$  is the largest soft open set contained in of  $(F, B)$ .

Consider example 3,  $(F_4, M)^\circ = \{(\{h_2\}, e_1), (\{h_1\}, e_2)\}$ .

**Definition 3.4** Let  $(G, \tau, M)$  be a soft rough topological space over  $\mathcal{T}$ , in which,  $\mathcal{T} = (G, M, R, F)$  is a rough soft formal context,  $G$  is an initial universe set,  $M$  is a set of attributes, and  $(F, B)$  is a soft set over  $G$ , the **associated soft set** with  $(F, B)$  denoted by  $(\overline{F}, B)$  and  $\overline{F}(\alpha) = \overline{F(\alpha)}$ , where  $\overline{F(\alpha)}$  is the closure of  $F(\alpha)$  in  $\tau_\alpha$  for each  $\alpha \in B$ .

**Proposition 3.3** Let  $(G, \tau, M)$  be a soft rough topological space over  $\mathcal{T}$ , in which,  $\mathcal{T} = (G, M, R, F)$  is a rough soft formal context, which  $G$  is an initial universe set,  $M$  is a set of attributes, and  $(F, B)$  is a soft set over  $G$ , then  $(\overline{\overline{F}}, B) = \overline{(\overline{F}, B)}$ , and if  $(\overline{F}, B)' \in \tau$ , then  $(\overline{F}, B) = \overline{(\overline{F}, B)}$ .

**Proof** For any  $\alpha \in B \subseteq M$ ,  $\overline{F(\alpha)}$  is the smallest closed set which contain  $F(\alpha)$ . Moreover if  $(\overline{F}, B) = (H, M)$ , then  $H(\alpha)$  is also closed set in  $(G, \tau_\alpha)$  containing  $F(\alpha)$ , so  $\overline{F(\alpha)} = \overline{F(\alpha)} \subseteq H(\alpha)$ , that is,  $(\overline{F}, B) \tilde{\subset} \overline{(\overline{F}, B)}$ . Furthermore, if  $(\overline{F}, B)' \in \tau$ , then  $(\overline{F}, B)$  is a soft closed set containing  $(F, B)$ , and  $(\overline{F}, B) \subseteq \overline{(\overline{F}, B)}$ , by the definition of soft closure of  $(F, B)$ , having  $(\overline{F}, B) = \overline{(\overline{F}, B)}$ .

**Definition 3.5** Let  $(G, \tau, M)$  be a soft rough topological space over  $\mathcal{T}$ , in which,  $\mathcal{T} = (G, M, R, F)$  is a



rough soft formal context,  $G$  is an initial universe set,  $M$  is a set of attributes, and  $(F, B)$  is a soft set over  $G$ ,  $x \in G$ , if there is a soft open set  $(F_1, B_1)$  over  $G$ , such that  $x \in (F_1, B_1) \tilde{\subset} (F, B)$ , then  $x$  is a **soft interior point** of  $(F, B)$  and  $(F, B)$  is the **soft neighborhood** of  $x$ .

Clearly, for any soft open set  $(F, B)$  over  $M$ , because  $\forall x \in \bigcap_{\alpha \in B} F(\alpha) \subseteq G \implies x \in F(\alpha) \implies x \in (F, B) \tilde{\subset} (F, B)$ ,  $(F, B)$  is a soft neighborhood of each point of  $\bigcap_{\alpha \in B} F(\alpha)$ .

#### 4 The properties of topology over the rough soft formal context

**Definition 4.1** Let  $(G, \tau, M)$  be a soft rough topological space over  $T$  in which,  $T = (G, M, R, F)$  is a rough soft formal context,  $G$  is an initial universe set,  $M$  is a set of attributes, and  $(F, B)$  is a soft set over  $G$ , and  $x \in G$ . we say  $x \in (F, B)$  (read as  $x$  **belongs to** the soft set  $(F, B)$ ) if  $x \in F(e)$  for all  $e \in B$ , and if there is some  $e \in B$ ,  $x \notin F(e)$ ,  $\forall x \in G$ , then  $x \notin (F, B)$ .

**Proposition 4.1** Let  $(G, \tau, M)$  be a soft rough topological space over  $T$ , in which,  $T = (G, M, R, F)$  is a rough soft formal context, which  $G$  is an initial universe set,  $M$  is a set of attributes, and  $(F, B)$  is a soft set over  $G$ , and  $x \in G$ , if  $x$  is a soft interior point of  $(F, B)$ , then  $x$  is also an interior point of  $F(\alpha)$  in  $\tau_\alpha$ , for each  $\alpha \in B$ .

**Proof** For all  $\alpha \in M$ , having  $F(\alpha) \subseteq G$ , if  $x \in G$  is a soft interior point of  $(F, B)$ , then  $\exists (F_1, B_1) \in \tau$ , such that  $x \in (F_1, B_1) \tilde{\subset} (F, B)$ , that is,  $x \in F_1(\alpha) \subseteq F(\alpha)$ . As  $F_1(\alpha) \in \tau_\alpha$ , so  $F_1(\alpha)$  is an open set in  $\tau_\alpha$ , and  $x \in F_1(\alpha)$ , hence,  $x$  is an interior point of  $F(\alpha)$  in  $\tau_\alpha$ .

**Proposition 4.2** Let  $T = (G, M, R, F)$  be a rough soft formal context, and  $(G, \tau, M)$  be a soft topological space over the rough soft formal context  $T = (G, M, R, F)$ , then

- (1) each  $x \in G$  has soft neighborhood;
- (2) if  $(F_1, B_1)$  and  $(F_2, B_2)$  are soft neighborhood of some  $x \in G$ , then  $(F_1, B_1) \cap (F_2, B_2)$  is also a soft neighborhood of some  $x$ ;
- (3) if  $(F_1, B_1)$  is a soft neighborhood of  $x \in G$ , and  $(F_1, B_1) \tilde{\subset} (F_2, B_2)$ , then  $(F_2, B_2)$  is also a soft neighborhood of some  $x$ .

**Proof**

(i) For all  $x \in G$ , having  $x \in \tilde{G}$ , and since  $\tilde{G} \in \tau$ , so  $x \in \tilde{G} \subseteq \tilde{G}$ , thus  $\tilde{G}$  is a soft neighborhood of  $x$ .

(2) Let  $(F_1, B_1)$  and  $(F_2, B_2)$  be the soft neighborhoods of some  $x \in G$ , then  $\exists (F_1', B_1'), (F_2', B_2') \in \tau$  such that  $x \in (F_1', B_1') \tilde{\subset} (F_1, B_1)$ , and  $x \in (F_2', B_2') \tilde{\subset} (F_2, B_2)$  now  $x \in (F_1', B_1')$ ,  $x \in (F_2', B_2')$

imply  $x \in ((F_1', B_1') \cap (F_2', B_2')) \tilde{\subset} ((F_1, B_1) \cap (F_2, B_2))$ . Thus  $(F_1, B_1) \cap (F_2, B_2)$  is also a soft neighborhood of some  $x$ ;

(3) Let  $(F_1, B_1)$  be a soft neighborhood of  $x \in G$ , and  $(F_1, B_1) \tilde{\subset} (F_2, B_2)$ , then  $\exists (F_1', B_1') \in \tau$  such that  $x \in (F_1', B_1') \tilde{\subset} (F_1, B_1) \tilde{\subset} (F_2, B_2)$ , so  $(F_2, B_2)$  is also a soft neighborhood of some  $x$ .





**Theorem 4.1** Let  $(G, \tau, M)$  be a soft rough topological space over  $T$ , in which,  $T = (G, M, R, F)$  is a rough soft formal context,  $G$  is an initial universe set,  $M$  is a set of attributes, and  $(F_i, B_i), B_i \subseteq M, i = 1, 2$  is a soft set over  $G$ . Then

- (i)  $\emptyset^\circ = \emptyset, \tilde{G}^\circ = \tilde{G}$ ;
- (ii)  $(F, B)^\circ \tilde{\subset} (F, B)$ ;
- (iii)  $(F, B)$  is a soft open set if and only if  $(F, B)^\circ = (F, B)$ ;
- (iv)  $((F, B)^\circ)^\circ = (F, B)$ ;
- (v)  $(F_1, B_1) \tilde{\subset} (F_2, B_2)$  implies  $(F_1, B_1)^\circ \tilde{\subset} (F_2, B_2)^\circ$ ;
- (vi)  $(F_1, B_1)^\circ \cup (F_2, B_2)^\circ \tilde{\subset} ((F_1, B_1) \cup (F_2, B_2))^\circ$ ;
- (vii)  $(F_1, B_1)^\circ \cap (F_2, B_2)^\circ = ((F_1, B_1) \cap (F_2, B_2))^\circ$ .

**Proof** By definition of soft interior point of  $(F, B)$ ,  $(F, B)^\circ = \bigcup_{(F_i, B_i) \in \tau} \{(F_i, B_i) \tilde{\subset} (F, B)\}$ , (i),(ii) are obvious.

And  $(F, B)^\circ$  is a soft open set,  $((F, B)^\circ)^\circ$  is the union of all soft open subsets in  $G$  contained in  $((F, B)^\circ)$ ,  $(F, B)^\circ \tilde{\subset} ((F, B)^\circ)^\circ$ , but  $((F, B)^\circ)^\circ \tilde{\subset} (F, B)^\circ$ , hence (iv) holds.

If  $(F, B)$  is a soft open set, then  $(F, B)$  is itself a soft open set over  $G$  which contains  $(F, B)$ , so  $(F, B)^\circ$  is the largest soft open set contained in  $(F, B)$ , and  $(F, B)^\circ = (F, B)$ ; Conversely, if  $(F, B)^\circ = (F, B)$ , by  $(F, B)^\circ$  is a soft open set,  $(F, B)$  is a soft open set, too. That is, (iii) holds.

(v) Suppose that  $(F_1, B_1) \tilde{\subset} (F_2, B_2)$ , since  $(F_1, B_1)^\circ \tilde{\subset} (F_1, B_1) \tilde{\subset} (F_2, B_2)$ .  $(F_1, B_1)^\circ$  is a soft open subset of  $(F_2, B_2)$ , so by definition of  $(F_2, B_2)^\circ$ , having  $(F_1, B_1)^\circ \tilde{\subset} (F_2, B_2)^\circ$ .

(vi) By (v),  $(F_1, B_1) \tilde{\subset} (F_2, B_2) \cup (F_1, B_1)$ ,  $(F_2, B_2) \tilde{\subset} (F_1, B_1) \cup (F_2, B_2)$  implies  $(F_1, B_1)^\circ \tilde{\subset} ((F_1, B_1) \cup (F_2, B_2))^\circ$ , and  $(F_2, B_2)^\circ \tilde{\subset} ((F_1, B_1) \cup (F_2, B_2))^\circ$ , by  $(F_1, B_1)^\circ \cup (F_2, B_2)^\circ$  is a soft open set, so  $(F_1, B_1)^\circ \cup (F_2, B_2)^\circ \tilde{\subset} ((F_1, B_1) \cup (F_2, B_2))^\circ$ .

(vii) By (v),  $(F_1, B_1) \tilde{\subset} (F_2, B_2) \tilde{\subset} (F_1, B_1)$ ,  $(F_1, B_1) \tilde{\subset} (F_2, B_2) \tilde{\subset} (F_2, B_2)$  implies  $((F_1, B_1) \cap (F_2, B_2))^\circ \tilde{\subset} (F_1, B_1)^\circ$ ,  $((F_1, B_1) \cap (F_2, B_2))^\circ \tilde{\subset} (F_2, B_2)^\circ$  implies  $((F_1, B_1) \cap (F_2, B_2))^\circ \tilde{\subset} (F_1, B_1)^\circ \cap (F_2, B_2)^\circ$ . Also, since  $(F_1, B_1)^\circ \tilde{\subset} (F_1, B_1)$  and  $(F_2, B_2)^\circ \tilde{\subset} (F_2, B_2)$  implies  $(F_1, B_1)^\circ \cap (F_2, B_2)^\circ \tilde{\subset} ((F_1, B_1) \cap (F_2, B_2))^\circ$ , so that  $(F_1, B_1)^\circ \cap (F_2, B_2)^\circ$  is a soft open subset of  $((F_1, B_1) \cap (F_2, B_2))^\circ$ , so  $(F_1, B_1)^\circ \cap (F_2, B_2)^\circ \tilde{\subset} ((F_1, B_1) \cap (F_2, B_2))^\circ$ . Thus,  $(F_1, B_1)^\circ \cap (F_2, B_2)^\circ = ((F_1, B_1) \cap (F_2, B_2))^\circ$ .

## 5 Conclusion

In this paper, we discuss the topology based on rough soft formal context. We give the definition of topology over the rough soft formal context. We give some topological definitions on the rough soft formal context, and discuss their properties. That is, we investigate the topological structure combining soft sets with rough sets and formal context, some different types of hybrid models are presented, which is topology over the rough soft formal context. That offers a new method and tool in data analysis. Therefore, we can deal with lots of data more easily and do more efficient decision in data mining, information system, human reasoning and so on.

There are some other topological properties such as separation axioms over the rough soft formal context, we will discuss them later.

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