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Fractal Modeling of Retinal Blood Vessel System

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Abstract

This paper proposes a novel approach to extract the main features of retinal blood vessel system. The purpose of the study is to evaluate the blood flow in the retinal blood vessel system using Darcy's law, Reynold's number, Poiseuille's law and Forchheimer equation. A new algorithm has been established to traverse through the retinal blood vessel system in a robust manner.

Key words:

Fractals; Retinal blood vessel system; Poiseuille's la ; Forchheimer equation.

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Introduction

In this paper complex geometrical (fractal) models of human retinal blood vessels have been proposed. The pressure drop has been found out according to Darcy's law [13]. In the recent years fractal analysis of blood vessel in different parts of human body has been described. The present work deals with fractal retinal blood vessel trees that have been classified and examined.

Fractals

A Fractal is an object which appears self-similar under varying degrees of magnification. Self-similarity is the major characteristic of the fractal objects [7]. Recent studies have attempted with some success to characterize certain parts of the body using fractal geometry where the retinal blood vessel system exhibits fractal characteristic. There are many definitions for fractal, among them

(a) "A fractal is a shape made of parts similar to the whole in some way".

(b) "A fractal is by definition for which the Hausdorff dimension strictly exceeds the Topological dimension"

Mandelbrot (1975) introduced the term "FRACTAL" to characterize spatial or temporal phenomena that are continuous but not differentiable. Fractal objects and processes are therefore said to display 'Self-invariant' (Self-similar or Self-affine) properties [5]. Fractal structures do not have a single length scale, while fractal processes (time-series) cannot be characterized by a single-time scale [10].

Structure of Retina

The retina is the light-sensitive tissue that lines the inside of the eye. The retina functions in a manner similar to film in a camera. The optical elements within the eye focus an image onto the retina of the eye, initiating a series of chemical and electrical events within the retina. Nerve fibers within the retina send electrical signals to the brain, which then interprets these signals as visual images.

Fractal model of blood vessel system is a certain geometrical simplification but it suffices for acceptable blood flow analysis [4]. This analysis permits understanding influence of hemodynamic forces.

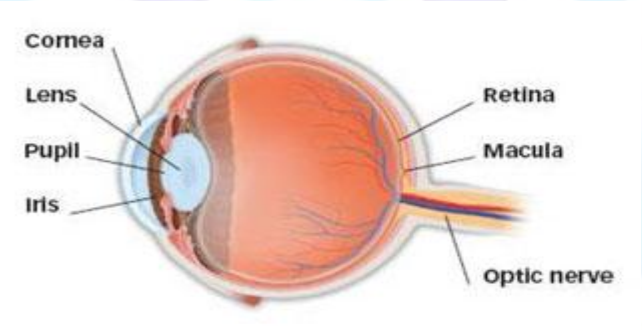

Fig-1: Anatomical structure of the eye

- **The Cornea**: The cornea is the transparent front part of the eye that covers the iris, pupil, and anterior chamber. The cornea, with the anterior chamber and lens, refracts light, with the cornea accounting for approximately two-thirds of the eye's total optical power. The cornea acts like a window which allows light to pass through several tissues in the eye before it arrives at the retina.
- **Pupil and Iris**: Light passes through the Pupil, the circular aperture is in the middle of the Iris (the colored tissue seen through the cornea). The pupil contracts in bright light to decrease the amount of light reaching the retina, thus preventing damage to the retina by excessive light or it dilates in dim light to ensure that enough light reaches the retina for vision to occur.
- **The Crystalline Lens**: The crystalline lens is a transparent, biconvex structure in the eye that, along with the cornea, helps to refract light to be focused on the retina. The lens, by changing shape, functions to change the focal distance of the eye so that it can focus on objects at various distances, thus allowing a sharp real image of the object of interest to be formed on the retina.
- **The Retina**: The retina surrounds most of the cavity of the eyeball.
- **Macula:** The **macula** or **macula lutea** (from Latin *macula*, "spot" + *lutea*, "yellow") is an oval-shaped highly pigmented yellow spot near the center of the retina of the human eye.
- **The optic nerve:** The optic nerve, also known as cranial nerve II**,** transmits visual information from the retina to the brain.
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- ## Modeling of the blood flow through fractals

Blood is a non-Newtonian fluid consisting of blood cells and blood plasma. Assumption of constant blood viscosity and homogeneity in whole vessels tree is necessary to estimate blood flow through blood vessel trees. During a normal flow in straight arteries blood behaves as a Newtonian fluid [8].

In real blood vessels system, vessel walls are elastic and change its diameters[6]. In this way resistance of blood vessel system is regulated. This process is known as autoregulation and corrects nutrition of all cells in human body [9-11]. Blood flow estimation assumes laminar flow for the entire fractal vascular tree. In large arteries a rhythmical contraction of a deviation from the normal of the laminar flow is a result of wave propogation. [2, 3, 12] (Fig.2)

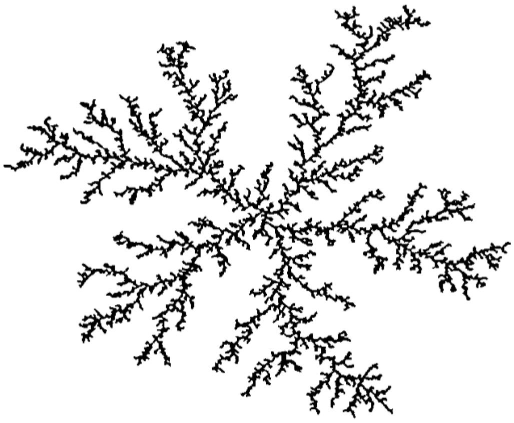

Fig – 2: Three Dimensional geometry of fractal vascular tree

Methods

Blood flow in the retinal blood vessel system has been mathematically calculated by the following methods. These results are very useful to investigate the defects in the above system.

Rate of flow (Q)

It is defined as the quantity of the fluid flowing per second through a section of a pipe or a channel. For an incompressible fluid the rate of flow is expressed as the volume of fluid flowing across the section per second. Mathematically the rate of flow is defined as

$$Q = A \times V \qquad \text{.....(1)}$$

where Q is the Rate of blood flow (m□ /sec), A is the cross-secional area (cm^2), V is the velocity of the blood.

Poiseuille's law

The quantity of blood that flows through a vessel in a given period of time i.e. per unit of time, is equal to the velocity of flow times the cross sectional area according to the following equations:

$$Q = V\pi a^2 \qquad \text{.....(2)}$$

$$V = (P_1 - P_2)a^2 / 8\eta L$$

Now equation (1) becomes

$$Q = \left[(P_1 - P_2)a^2 / 8\eta L\right]\pi a^2$$

$$Q = \frac{\pi * a^4 \Delta P}{8\eta L} \qquad \text{.....(3)}$$

where ΔP is the pressure difference $(P_1 - P_2)$, η is the Viscosity of the Blood (Poises), L is the Length of the Vessel (Cms). The equation (2) is known as Hagen-Poiseuille's equation. This equation shows that the rate of blood flow is directly proportional to the fourth power of the radius of the vessel and also illustrates that the diameter of a vessel plays a great role in determining the rate of blood flow.

The most effective factor controlling blood flow is radius of the blood vessel. High blood pressure can be caused by narrowing blood vessel and is reduced by relaxing the smooth muscle tension that controls the blood vessel radius. This process is known as an auto regulation.

Darcy's law

In the case of abnormalities, Darcy's law can be used to find the permeability.

$$Q = -\frac{\kappa A \Delta P}{\eta L} \qquad \text{.....(4)}$$

where Q is the rate of flow, κ is the permeability, A, is the area of the cross-section, L is the length of the blood vessel, η is the viscosity of the blood, and ΔP is the pressure difference. The equation(4) is known as the Darcy's law.

Now equation (4) becomes $$\kappa = -\frac{Q\eta L}{A\Delta P} \qquad \text{....(5)}$$

The equation (5) determines the complexity in the blood flow at various parts of the cardiovascular system. The negative sign is needed because fluids flow from high pressure to lower pressure. So if the change in the pressure is negative then the flow will be positive (Table2).

Pressure Drop

Pressure drop is a positive function depending on the blood flow, which passes through the different parts of the retinal blood vessel system. Pressure drop have been calculated according to Poiseuille's law, the pressure drop P for rate of flow Q through a blood vessel with radius *a* over a length L is

$$P = \frac{8\eta QL}{\pi a^4} \qquad \text{.....(6)}$$

Pressure drop maintains the energy dissipation rate of the flow of blood for the various parts of the system (Table2).

Reynold's number

When normal laminar blood flow becomes turbulent flow

- The rate of blood flow (i.e) the velocity of flow, is high
- It passes by an obstruction in a vessel (as in case of compression by cuff of Sphygmomanometer).
- It makes a sharp turn.
- It passes over a rough surface

Turbulence is also related to the diameter of the vessel and the viscosity of the blood which can be expressed by ratio of inertia to viscous forces.

$$R_e = \frac{V * L}{\eta} \qquad \text{.....(7)}$$

where R_e is the Reynold's number, L is the length of the blood vessel, V is the Average velocity and η is the Viscosity of the blood. This is the most important dimensionless number; it describes the fluid flow regime. If R_e is higher, there is a greater probability of the turbulence (Table2).

Inlet Section

The blood flow in the blood vessel is mainly concerned with Inlet section. The length of the section is proportional to vessel diameter, Reynold's number and coefficient

$$l_e = \lambda d R_e \qquad \text{.....(8)}$$

Where l_e is the inlet section, $\lambda = 0.056$ a value from Navier-Stokes equation, d diameter of the blood vessel and R_e Reynold's number

The above factors has been analysed for normal patient and is given in the Table 2.

Algorithm

The given algorithm perceives the blood vessels of the retina in a regular manner which mainly circulates the blood to retina in an orderly manner.

Name: Recursive algorithm for the inorder traversal of a binary tree.

Input: A binary tree represented by the arrays LEFTSON and RIGHTSON

Output: An array called NUMBER such that (number of i) NUMBER[i] is the inorder number for the vertex i.

Method: In addition to LEFTSON , RIGHTSON and NUMBER, the algorithm makes use of the variable COUNT which contains the inorder number to be assigned to a vertex. The initial value for the COUNT is 1.

The parameter VERTEX is initially the root.

The procedure is recursively used as follows:

Procedure INORDER (VERTEX)

Begin

Step 1: If LEFTSON [VERTEX] ≠ 0 then

INORDER (LEFTSON [VERTEX]);

Step 2: NUMBER [VERTEX] ←COUNT;

Step 3: COUNT←COUNT+1;

Step 4: If RIGHTSON [VERTEX] ≠0tehn

INORDER (RIGHTSON [VERTEX]);

End

Time complexity: Time complexity of this recursive algorithm of inorder traversal is order of n [1]. i.e.

$$T(n) = O(n)$$

The above algorithm has been programmed and run by C++.

Numerical Simulation Results

The human retinal blood vessel system features a tremendous variety of components with different geometrical dimensions and mechanical properties. A synthetic view is given in the following Table.1.

Table 1 Data Report of Normal eye

Vessels	Radius (r) (mm)	Total Cross Section (A) (mm^2)	Average Velocity (V) (μl/min)	Length (L) (mm)
Superior Temporal Artery	0.052±0.009	0.104±0.018	86±10	0.2
Inferior Temporal Artery	0.0545±0.0095	0.109±0.019	79±13	0.2
Superior Temporal Vein	0.0645±0.0125	0.129±0.025	89±12	0.2
Inferior Temporal Vein	0.0685±0.010	0.137±0.020	74±12	0.2
Neuro Retinal Rim	1.015±0.205	2.03±0.41	80±12	0.2

The viscosity of the blood is 5 Poise. For example, the blood vessel Superior temporal artery has been shown.

1) Area of the cross section

$$C = \pi r^2$$
$$= \pi \times (0.052 \pm 0.009)^2$$
$$= (8.494867 \times 10^{-3} \pm 2.54469 \times 10^{-4}) mm^2$$

2) Rate of flow

$$Q = A \times V$$
$$= (0.104 \pm 0.018) \times (86 \pm 10)$$
$$= (8.944 \pm 0.18) mm^3 / \sec$$

3) Reynolds number

$$R_e = \frac{V * L}{\eta}$$
$$= \frac{(86 \pm 10) \times 0.2}{5}$$
$$= (3.44 \pm 0.4)$$

4) Inlet Section

$$l_e = \lambda d R_e$$
$$= 0.056 \times (0.104 \pm 0.018) \times (3.44 \pm 0.4)$$
$$= (2.003456 \times 10^{-2} \pm 4.032 \times 10^{-4}) \mu m$$

5) Pressure drop (Poiseuille's Law)

$$P = \frac{8 \eta Q L}{\pi r^4}$$
$$= \frac{8 \times 5 \times (0.730558522 \pm 2.54469 \times 10^{-3}) \times 0.2}{\pi \times (0.052 \pm 0.009)^4}$$
$$= (254437.8698 \pm 987654.3210) mm^3 / \sec$$

6) Pressure drop (Forchheimer Equation)

$$P = \rho \frac{\beta}{\pi r^4}$$
$$= 5 \times \frac{0.639}{\pi \times (0.052 \pm 0.009)^4}$$
$$= (139093.7498 \pm 155006871.9) mm^3 / \sec$$

7) Darcy's Law

$$\kappa = -\frac{Q \eta L}{A \Delta P}$$
$$= -\frac{(0.730558522 \pm 2.54469 \times 10^{-3}) \times 5 \times 0.2}{(0.104 \pm 0.018) \times (254437.8698 \pm 987654.3210)}$$
$$= -(2.760831624 \times 10^{-5} \pm 1.431388 \times 10^{-7}) mm^3 / \sec$$

8) Total blood flow

$$Q = \pi \Delta P d^4 / 128 \eta L$$
$$= \pi (254437.8698 \pm 987654.3210)(0.104 \pm 0.018)^4 / 128 \times 5 \times 0.2$$
$$= (0.730558522 \pm 2.54469 \times 10^{-3}) dynes / mm^2$$

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Table 2. Analysis of Data for other parts of the retinal blood vessel system of normal eye.

Vessels	Area of the cross section (C) (mm^2)	Rate of flow (Q) (mm^3/sec)	Reynolds number(R_e)	Inlet Section(l_e)(μm)	Pressure drop (P) (mm^3/sec) (Poiseuill's Law)	Pressure drop (P) (mm^3/sec) (Forchheimer Equation)	Darcy's Law (κ) (mm^2/sec)	Total blood flow (Q) mm^3/sec
Superior Temporal Artery	$8.494867\times10^{-3} \pm 2.54469\times10^{-4}$	8.944 ± 0.18	3.44 ± 0.4	$2.003456\times10^{-2} \pm 4.032\times10^{-4}$	254437.8698 ± 987654.321	139093.7498 ± 155006871.9	$-(2.760831624\times10^{-5} \pm 1.431388\times10^{-7})$	$0.730558522 \pm 2.54469\times10^{-3}$
Inferior Temporal Artery	$9.331315579\times10^{-3} \pm 2.83528737\times10^{-4}$	8.611± 0.247	3.16± 0.52	$1.928864\times10^{-2} \pm 5.5328\times10^{-4}$	212776.7023± 1152354.571	115274.9601± 124860930.9	$-(3.178479369\times10^{-5} \pm 1.683451876\times10^{-7})$	$0.73717393 \pm 3.685873582\times10^{-3}$
Superior Temporal Vein	$1.306981\times10^{-2} \pm 4.908738521\times10^{-4}$	11.481± 0.3	3.56± 0.48	$2.571744\times10^{-2} \pm 6.72\times10^{-4}$	171143.5611± 614400	58760.04656 ± 41656323.54	$-(5.268767492\times10^{-5} \pm 3.834951969\times10^{-7})$	$1.163213165 \pm 5.890486225\times10^{-3}$
Inferior Temporal Vein	$1.4741138\times10^{-2} \pm 3.141592654\times10^{-4}$	10.138± 0.24	2.96± 0.48	$2.270912\times10^{-2} \pm 5.376\times10^{-4}$	126165.4856±959999.9999	46191.14245± 101700008.6	$-(6.31104976\times10^{-5} \pm 1.963495408\times10^{-7})$	$1.090844221 \pm 3.769911184\times10^{-3}$
Neuro Retinal Rim	3.236547292 ± 0.132025431	162.4± 4.92	3.2± 0.48	0.363776± 0.0110208	621.2235191± 2284.354551	0.958201443± 575.8449275	$-(0.205318468 \pm 1.691575838\times10^{-3})$	258.9237833± 1.584305175

Conclusion

The blood flow analysis in the blood vessels have been numerically calculated for the parameters such as area of the cross section, rate of flow, Pressure drop, Reynold's number, Darcy's law and inlet section for a normal patient, in our proposed system .Darcy's law is a new concept which provides us the pressure variation due to inflammation and gives us the accurate result of the affected blood vessels of the retinal blood vessel system in an effective manner. This numerical simulation helps us in understanding the blood circulation in the blood vessels of the retina. The recursive strategy is a useful tool to go through the system in an effective manner.

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