# ANALYSIS OF MODIFIED COSH WINDOW FUNCTION AND PERFORMANCE EVALUATION OF THE FIR FILTER DESIGNED USING WINDOWING TECHNIQUES

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## ABSTRACT

Filters are used in electronic circuits to remove the unwanted frequency components from desired signals. A digital filter basically provide high attenuation to the unwanted ones and offer very low or ideally zero attenuation to desired signal components when it's impulse response is adjusted as per requirement. For ideal filters, the length of such an impulse response is infinite and also the filter will be non-causal and unrealizable. So, we need to truncate this infinite impulse response to make it finite.

For this truncation, we use window functions. Using window functions, we obtain a finite impulse response or simply FIR filter. The shape of a window in time domain decides the characteristics of resultant filter in frequency domain. Several window functions are available in literature. For the present work we have choosen the three parameter Cosh window for truncation of infinite impulse response. It is also called as modified Cosh window because it has been obtained by inserting a third parameter in the basic 2-parameter Cosh window function.

The main goal of this work is to study this modified Cosh window and design a digital low pass FIR filter using the same in MATLAB. First of all the properties of window function are described and frequeny domain responses of window function is obtained. Then FIR filter is analyzed using window design method and it's characteristics have also been studied in frequency domain.

**Keywords:** FIR filter, Cosh, MSA, FSA, LPF.

## 1. INTRODUCTION

Digital filters are the most important and frequently used elements in digital signal processing applications. The input signal is sampled and an analog-to-digital converter turns the signal into a stream of numbers. A computer program running on a CPU or a specialized DSP (or less often running on a hardware implementation of the algorithm) calculates an output number stream. This number stream is then filtered by convolving it with the impulse response of the filter. The resulting number stream output can be converted into an analog signal by passing it through a digital-to-analog converter.

Digital filters are classified as finite impulse response (FIR) and infinite impulse response (IIR) filters based on the duration of their impulse response. A finite impulse response (FIR) filter is a type of a signal processing filter whose impulse response (or response to any finite length input) is of finite duration, because it settles to zero in finite time. The advantages of FIR filter are that these are most popular, always stable and these have linear phase. The main disadvantage of FIR filters over IIR filters is their implementation complexity when the filter order is very large. The implementation of FIR filters with non-recursive techniques guarantees stability.

For a discrete-time FIR filter, the output is a weighted sum of the current and a finite number of previous values of the input. The operation is described by the following equation, which defines the output sequence *y[n]* in terms of its input sequence *x[n]*:

$$y[n] = b_0 x[n] + b_1 x[n-1] + \cdots b_N x[n-N]$$

$$= \sum_{i=0}^{N} b_i x[n-i]$$

Where, *x[n]* is the input signal, *y[n]* is the output signal, $b_i$ are the filter coefficients, also known as tap weights, that make up the impulse response and *N* is the filter order.

## 2. WINDOW DESIGN METHOD

The desired frequency response of any digital filter is periodic in frequency and can be expanded in Fourier series, i.e.

$$H_d(e^{jw}) = \sum_{n=-\infty}^{\infty} h_d(n) e^{-jwn}$$

where

$$h(n) = \frac{1}{2\pi} \int_0^{2\pi} H(e^{jw}) e^{jwn} dw$$

The coefficients of the series *h(n)* are identical to the impulse response of a digital filter. Now, this impulse response is of infinite duration and the filter is non-causal and unrealizable. The infinite duration impulse response is converted into a finite duration impulse response; by truncating the infinite series. This results in undesirable oscillations in the pass band and stop band of the digital filter. This is due to the slow convergence of fourier series near the points of discontinuity. The undesirable oscillations are reduced by using a set of time-limited weighting functions, *w (n)*, referred to as window functions, to modify fourier series coefficients. A major effect of windowing is that the discontinuities in $H(e^{jw})$ are converted into transition bands between values on either side of the discontinuity. The width of these transition bands depends on the width of the main lobe of $W(e^{jw})$.

## 3. MODIFIED COSH WINDOW FUNCTION AND FILTER DESIGNING

Modified Cosh window is simply a 3-parameter Cosh window. The basis function for this window is hyperbolic cosine function. Mathematically, the modified Cosh window is given as:

$$w[n] = \left( \frac{\cosh\left( \alpha \sqrt{1 - \left( \frac{2n}{N-1} \right)^2} \right)}{\cosh(\alpha)} \right)^{\rho}$$

$$0 \le n \le N-1$$

Where, *N* is the length of window function; $\alpha$ & $\rho$ are shape changing Parameters for window.

Now, in time domain this window function looks like as shown in figure 1 (For $\alpha = 2$, $\rho = 2$ and $N = 51$).

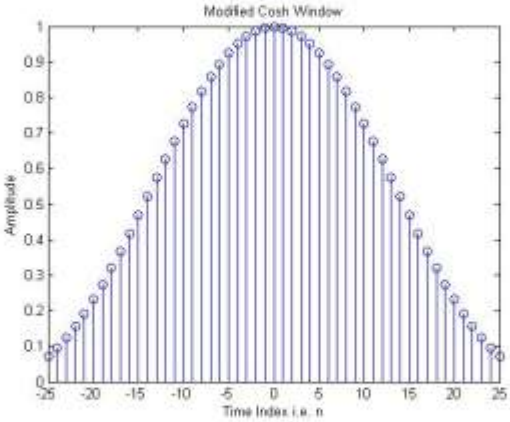


**Fig. 1 Modified Cosh window function with $\alpha = 2$ , $\rho = 2$ and $N = 51$**

The spectrum of this modified Cosh window function is as shown in the figure 2. From figure 2, we can find the main lobe width, ripple ratio and side lobe roll off ratio as follows:

**Ripple ratio = Maximum side lobe amplitude in dB – Main lobe amplitude in dB = S1 dB**

**Side lobe Roll off Ratio = Maximum side lobe amplitude in dB- Minimum side lobe amplitude in dB =S1 – SL dB**

**Main lobe width = 2Wr**

**And, Normalized main lobe width i.e. D = 2Wr (N-1) rad/sec**

The impulse response of an ideal low pass filter (LPF) is sync function of infinite length. To design an FIR filter we have to truncate this infinite impulse response with the help of a window function.

Then the resulting filter will be having a finite length and non-ideal frequency domain characteristics. In this way, we design a LPF using window functions.

Figure 3 and 4 shows the sync function and the frequency domain response of a LPF designed using window functions.

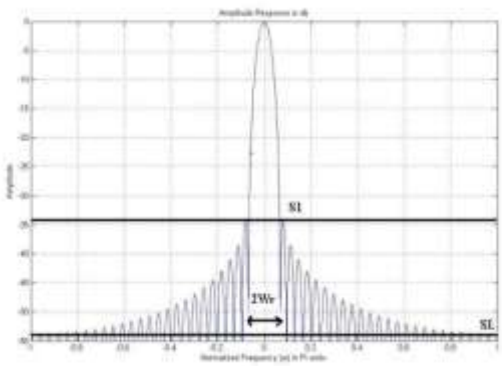


**Fig. 2 Frequency domain characteristics of modified Cosh window**

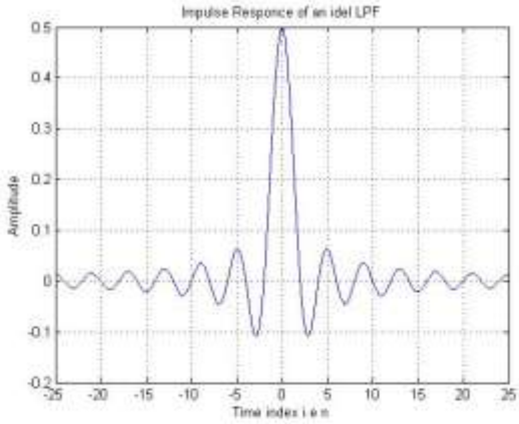


**Fig.3 Ideal sync pulse, impulse response of a LPF**

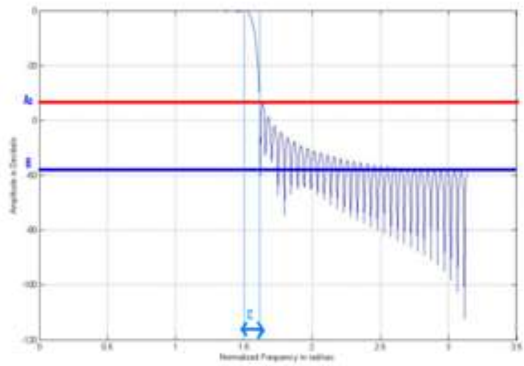

**Fig.4 frequency domain response of a LPF**

From the figure 4, we can find the following parameters for a LPF:

**Minimum stop band attenuation i.e. MSA = $A_s$ dB**

**Far-end stop band attenuation i.e. FSA = B dB**

**Transition width i.e. Δw = C rad/sec**

All these parameters form the basis of comparison of low pass filters designed using different window functions.

## 4. RESULTS & DISCUSSIONS

As we have analyzed the performace of an FIR filter designed using modified Cosh window.So first of all, the plots related to window function are being shown and then plots related to LPF are discussed.

### 4.1 BASIS FUNCTIONS OF WINDOW FUNCTIONS

The Basis function for Kaiser window is modified bessel function of First kind and zero order i.e. $I_0(x)$. This function has been plotted in figure 5 with $Cosh(x)^{\rho}$ for different values of $\rho$ .

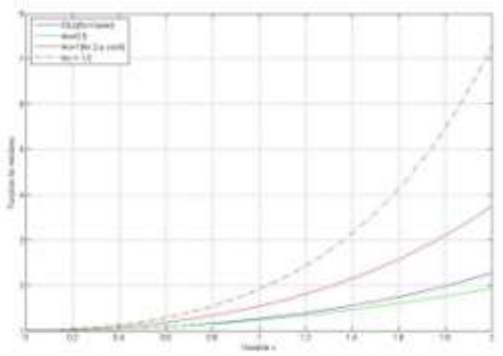

**Fig.5 Plot for Basis Functions for windows**

From this plot, It can be seen that the shapes of $Cosh(x)^{\rho}$ resembles with $I_0(x)$ for different values of $\rho$ . So because of this shape symmetry, $Cosh(x)^{\rho}$ is used as basis function for modified Cosh window as $I_0(x)$ has been used as basis function for Kaiser window.

## 4.2 MODIFIED COSH WINDOW IN TIME AND FREQUENCY DOMAIN

Next, different shapes for the modified cosh window in time domain and their respective spectrums in Frequency domain are being plotted. We know that $\alpha$ and $\rho$ are shape changing parameters for this window . So first of all, $\alpha$ is kept constant and $\rho$ is being changed. We are taking the length of window function i.e $N = 51$.

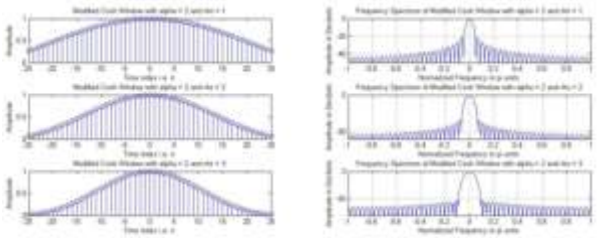

**Fig. 6 Modified Cosh window(s) and their respective spectrums when $\alpha$ =2, $N$= 51 and $\rho$ is varying**

From Figure 6, it can be seen that for a given value of $\alpha$ , as $\rho$ changes, the shape of the window function changes in time domain and correspondingly the main lobe width and the side lobe level for windows spectrum also gets changed. From the above plot, we calculated the following data values:

**Table 1 Ripple ratio, Side lobe roll off ratio and Normalized main lobe width for fixed $\alpha$**

| $\alpha = 2$ | | | |
|---|---|---|---|
| $\rho$ | **Ripple Ratio(db)** | **Side lobe Roll of Ratio(db)** | **Normalized Main lobe width(rad/sec)** |
| 1 | 21.63 | 21.87 | 5.2 |
| 2 | 34.24 | 19.55 | 7.0 |
| 3 | 52.69 | 12.01 | 9.2 |

For given values of $\alpha$ and $N$, when $\rho$ increases, we can summarize the above results as follows:

1. Main lobe width increases which results to an increase in transition width of finally designed Filter.

2. As the power difference between side lobe (next to main lobe) and the lobe at $w = \pi$ decreases, it means side lobe roll off ratio decreases.

3. Power level of side lobe (next to main lobe) decreases which results to an increase (in magnitude) in ripple ratio. As ripple improves for window function, MSA of finally designed filter gets improved.

So the value of this parameter $\rho$ is a crucial factor while designing a filter.

In the next step, $\rho$ is kept constant and $\alpha$ is being changed. Again we are taking the length of window function i.e $N = 51$.

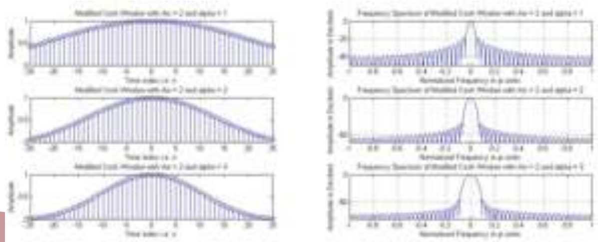

**Fig. 7 Modified Cosh window(s) and their respective spectrums when $\rho = 2$, $N$= 51 and $\alpha$ is varying**

From figure 7, it can be seen that for a given value of $\rho$ , as $\alpha$ changes, the shape of the window function changes in time domain and correspondingly the main lobe width and the side lobe level for windows spectrum also gets changed. From the figure 7, we calculated the data values as shown in table 2.

**Table 2 Ripple ratio, Side lobe roll off ratio and Normalized main lobe width for fixed $\rho$**

| $\rho = 2$ | | | |
|---|---|---|---|
| $\alpha$ | **Ripple Ratio (db)** | **Side lobe Roll of Ratio(db)** | **Normalized Main lobe width(rad/sec)** |
| 1 | 19.19 | 20.65 | 4.8 |
| 2 | 34.24 | 21.55 | 7.0 |
| 3 | 46.04 | 25.54 | 9.2 |

For given values of $\rho$ and $N$, when $\alpha$ increases, we can summarize the above results as follows:

1. Main lobe width increases which results to an increase in transition width of finally designed filter.

2. As the power difference between side lobe (next to main lobe) and the lobe at $w = \pi$ increases, it means side lobe roll off ratio increases.

3. Power level of side lobe (next to main lobe) decreases which results to an increase (in magnitude) in ripple ratio. As ripple ratio improves for window function, MSA of finally designed filter gets improved.

We conclude that the value of this parameter $\alpha$ is also a crucial factor while designing a filter.

## 4.3 LPF RESPONSE IN FREQUENCY DOMAIN

The infinite impulse response(s) of ideal LPF is then truncated with the help of four different modified Cosh window functions to design FIR type LPFs. Input conditions for designing are: $w_c = 0.5pi$, $\alpha = 2$, Window length i.e. $N = 127$ & $\rho = 1, 2, 3$ *and* $4$ for different cases.

Normalized amplitude spectrums (in frequency domain) of designed LPFs are as shown in fig.8.

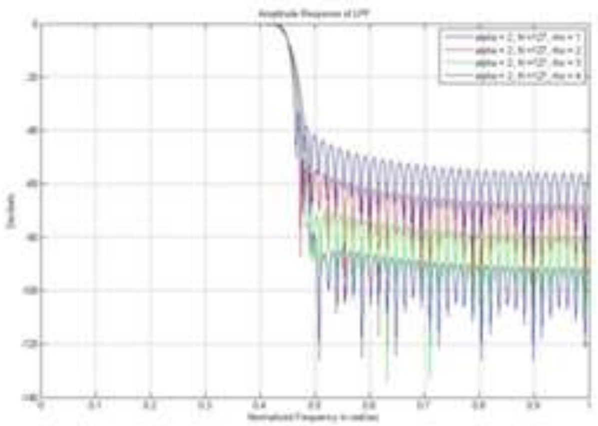

**Fig. 8 Normalized amplitude spectrums of LPFs**

From this plot, we can say that for given values of $\alpha$ and *N*, as $\rho$ changes, the transition width $(\Delta w)$ and side lobe level also gets changed. Following data values are obtained from figure 8.

**Table 3 Transition width, MSA and FSA calculations**

| $\rho$ | $\Delta w$ ( in rad /sec) | MSA ( in -db) | FSA (in -db) |
|---|---|---|---|
| 1 | 0.102 | 33.23 | 56.60 |
| 2 | 0.150 | 50.88 | 68.32 |
| 3 | 0.224 | 70.50 | 80.19 |
| 4 | 0.274 | 86.43 | 92.12 |

For given values of $\alpha$ and *N*, when $\rho$ increases, we can summarize the above results as follows:

1. Transition width $(\Delta w)$ of filter increases.

2. Minimum stop band attenuation (MSA) increases (in magnitude) or improves.

3. Far-end stop band attenuation (FSA) increases (in magnitude) or improves.

## 4.4 PLOT BETWEEN $\rho$ AND *MSA*

To show the relationship between $\alpha$ , $\rho$ and *MSA*, we have obtained data values from Normalized amplitude spectrums of LPFs designed using modified Cosh window. Input conditions were: $N$ = 127, $\alpha$ = *2* and *3* (For Different cases), $\rho$ is varied from 1 to 10 (for a given $\alpha$ ).

Then *MSA* is calculated for different LPFs. Data values obtained are listed in Table 4.

Figure 9 is plotted by using data values from table 4.

Following points may be concluded from figure 9:

1. As $\alpha$ increases then MSA increases (in magnitude) or improves.

2. For a given value of $\alpha$ , as $\rho$ increases then MSA increases (in magnitude) or improves.

The significance of this relationship is that in some cases, if it is not possible to change the value of one parameter ($\alpha$ or $\rho$ ), but a different amount of MSA is required, then other parameter ($\alpha$ or $\rho$ ) may be varied to change MSA. Moreover, these curves form the basis of window's design equations.

**Table 4. $\rho$ Vs. MSA for modified Cosh window with window length i.e. *N = 127***

| | $\alpha = 2$ | $\alpha = 3$ |
|---|---|---|
| $\rho$ | MSA ( in db) | MSA ( in db) |
| 1 | -33.23 | -42.14 |
| 2 | -50.88 | -72.11 |
| 3 | -70.50 | -92.26 |
| 4 | -86.43 | -111.20 |
| 5 | -98.19 | -137.10 |
| 6 | -109.90 | -162.60 |
| 7 | -123.40 | -183.80 |
| 8 | -137.20 | -210.40 |
| 9 | -147.90 | -225.80 |
| 10 | -161.00 | -249.50 |

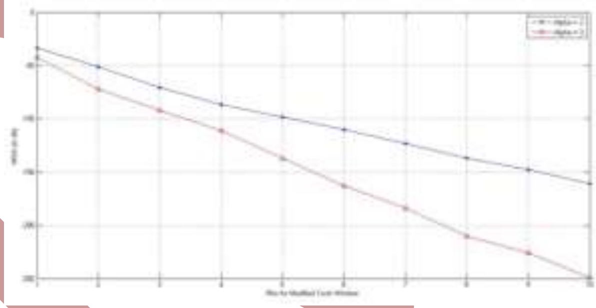

**Fig. 9 Relationship between $\rho$ and MSA for LPFs designed using modified Cosh window with N = 127**

## 5. CONCLUSIONS

Our simulation results show that for a given value of $\alpha$ and *N*, as $\rho$ increases, then main lobe width of the window's spectrum increases, ripple ratio increases (in magnitude) and side lobe roll off ratio decreases. For a given value of $\rho$ and *N*, as $\alpha$ increases, then main lobe width of the window's spectrum increases, ripple ratio increases (in magnitude) and side lobe roll off ratio increases.

Similarly, from the analysis of filter spectrum, we observed that for given values of $\alpha$ and *N* when $\rho$ increases, transition width $(\Delta w)$ of filter increases, minimum stop band attenuation (MSA) improves and far-end stop band attenuation (FSA) improves.

For a given value of $\rho$ , as $\alpha$ increases then minimum stop band attenuation (MSA) improves which can be useful for several DSP applications.

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