# Design of IIRFilter using Remez Algorithm

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## ABSTRACT

In this paper, we present a numerical method for the equiripple approximation of Impulse Infinite Response digital filters. The proposed method is based on the formulation of a generalized eigenvalue problem by using Rational Remez Exchange algorithm. In this paper, conventional Remez algorithm is modified to get the ratio of weights in the different bands exactly. In Rational Remez, squared magnitude response of the IIR filter is approximated in the Chebyshev sense by solving for an eigenvalue problem, in which real maximum eigenvalue is chosen and corresponding to that eigenvectors are found, and from that optimal filter coefficients are obtained through few iterations with controlling the ratio of ripples. The design algorithm is computationally efficient because it not only retains the speed inherent in the Remez exchange algorithm but also simplifies the interpolation step.

## Keywords

IIR digital filter, chebyshev approximation, rational remez algorithm, ripple ratio

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## 1. INTRODUCTION

For designing FIR filters, the Remez algorithm is the well-known method that guarantees optimality in the chebyshev sense. On the other hand, for an equiripple IIR filter design, the analog filter theory can be directly used [1]-[5]. By transforming the elliptic filter to z-domain, an optimal filter that realizes equiripple both in pass-band and stop-band is easily obtained. Though Remez algorithm is equally efficient algorithm but in its conventional low pass filter design, for example, when different weights are given for its pass-band and stop-band, one need to iteratively design the filter by trial and error to achieve the ratio of weights exactly. However this classical approach has critical limitations:

1. The order of denominator and numerator must be same.
2. It cannot handle multiband filters with different attenuation, etc.

To address these problems, many approaches for direct design of IIR filters have been proposed. The rational Remez algorithm is one such approach, which minimizes the Chebyshev norm of the squared magnitude response $|H(e^{jw})|^2$ of the rational functions and made it possible to directly control the ripples. The method iteratively solves the eigenvalue problem with controlling the ratio of ripples. [6]

In a method of iteratively linearizing the nonlinear constraints required in the nonlinear programming problem and applying the Rational Remez algorithm to the amplitude error function between the desired and designed frequency responses is an efficient approach, but it is not guaranteed that the convergent solution can be obtained.

In the next section, a brief review of the rational remez is discussed. Finally, some numerical examples are presented. It is obtained that the design method using the proposed method is better than that conventional method.

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## 2. PROBLEM FORMULATION

Modified Remez aims to obtain equiripple IIR filter with specified weighting function, α.

The transfer function of (M, N)$^{th}$order IIR filter is

$$H(e^{jw}) = \frac{N(w)}{D(w)} = \frac{\sum_{n=0}^{N} b_n \cos(nw_i)}{\sum_{m=0}^{M} a_m \cos(mw_i)} \qquad (1)$$

Where$a_o$is normalized to 1.[7]

Let D(w) be the desired magnitude response, then chebyshev norm is defined as

$$H(e^{jw}) - D(w) = \max \omega\, H(e^{jw}) - D(w) \qquad (2)$$

But due to non-linearity in the Chebyshev norm of the rational function, it requires numerical optimization techniques to solve (1) which may converge but needs a lot of computations. So to avoid numerical optimization, squared filter G (z) is used instead of H(z),

$$G(z) = H(z^{-1})H(z) \qquad (3)$$

Hence, the design problem is converted to

$$\max w\, [\min a, b\, \|G(w) - (D(w)^2\|] \qquad (4)$$

It is known that the alteration theorem holds for G (z).

**Alteration Theorem:** If a (M, N)th order rational function G(z) satisfies the alteration theorem, then G(w) is the best approximation to D(w)

1. $G(w_i) - D(w_i) = \frac{(-1)^i \delta}{W(w)}$ for

$$w_i = 0 \le w_1 < w_2 \ldots.. < w_{M+N+2}) \qquad (5)$$

2. $\delta = \max w \,\| G(w)\text{-}D(w)\|$ (6)

In the conventional Remez method, above 2 steps are iterated to get the equiripple response. The steps may be preceded as follows:

1. Solve (5) to get frequency points ω.
2. ω is replaced by the frequency points at which $G(w_i) - D(w_i)$ has local maxima.

The same process is iterated again and again until the frequency points are unchanged. [2]

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## 3. RATIONAL REMEZ ALGORITHM

### 3.1 Conventional Method

The transfer function of (M, N)$^{th}$order IIR filter is

$$H(e^{jw}) = \frac{N(w)}{D(w)} = \frac{\sum_{n=0}^{N} b_n cos(nw_i)}{\sum_{m=0}^{M} a_m \cos(mw_i)}$$

where$a_o$is normalized to 1. With transfer function H(z), D(w) being the desired response, chebyshev norm is defined as

$$\min a, b\, H(e^{jw}) - D(w) \qquad (8)$$

Hence, error function, E (w) is evaluated as,

$$E(w) = W(w)[H(e^{jw}) - D(e^{jw}) \qquad (9)$$

Where, W(w) is the weighting function, H(w) is the obtained frequency response.[7]
Substituting $H(e^{jw})$ in (9), E (w) can be generalized to the form:

$$N(w) - H_d(e^{jw})D(w) = \delta \frac{e^{(j\theta_e w)}}{W(w)} D(w)$$

And can be rewritten in Matrix form as:

$$Pb - Qa = \delta\, Ra \qquad (10)$$

Where,

$$\delta = \text{eigenvalue}$$
$$a = [a_0, a_1, \dots, a_M]^T$$
$$b = [b_0, b_1, \dots, b_n]^T$$
$$P_{(i,j)} = \cos(jw_i)$$
$$Q_{(i,k)} = D(w_i)\cos(kw_i)$$
$$R_{(i,k)} = \frac{-1^{i+1}}{W(w_i)} \cos(mw_i)$$

And,$i = 1,2, \dots\dots, M + N + 2$
$j = 0,1,2, \dots, N$
$k = 0,1,2, \dots\dots, M$

After some manipulations, eq (10) is simplified to the form

$$T = [P - Q]^{-1}[0\, R] \qquad (11)$$

The error δ and the coefficient matrix a, b are given by an inverse of the largest eigenvalues and its corresponding eigenvectors of T, respectively. [6]

## 3.2 Modified Method

Since, our aim is to design an equiripple IIR filter with specified weight, α. The above estimation works well but not when more precise ripple control is required. So, a modification in rational remez algorithm is made for the weights W (w) and ripples obtained depends on weight. Thus to control the weights of H(z), weights of G(z) are updated according to (12).

$$W_k(w) = \frac{\alpha^2 \delta_p^{k-1}}{4(M - \delta_p^{k-1})} \text{for, } \omega \text{ belongs to pass band} \qquad (12)$$

$$= 1 \qquad \text{for, } \omega \text{ belongs to stop band}$$

Resulting response obtained best approximates the desired filter characteristics with desired weighting function, W (w). [6]
Modified Remez Algorithm may be summarized as:

**STEP I:** With the specifications (N, M, $w_p$, $w_s$, W (w)) provided, initial frequency points $w_{i\,i=0}^{M+N+2}$ are specified.

**STEP II:** Initial pass-band ripple $\delta_p^{\ 0}$ is estimated.

**STEP III:** Solve Eigenvalue problem to find filter coefficients

**STEP IV:** Find local maxima of error function, E (w).

**STEP V:** Set extremal points to new frequency points $w_{i\,i=0}^{M+N+2}$ where local extrema is obtained.

**STEP VI:** If frequency points are unchanged, then stop. Otherwise go to Step III.

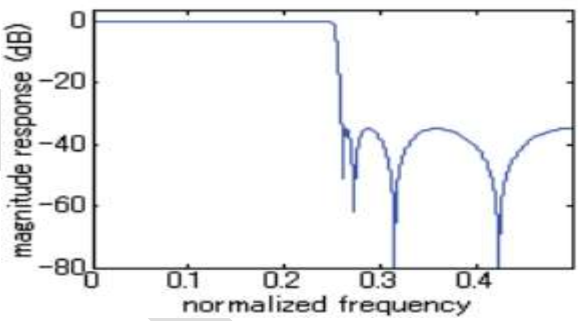
## 3.3 DESIGN EXAMPLES

In this section, one numerical example is presented to demonstrate the proposed algorithm.

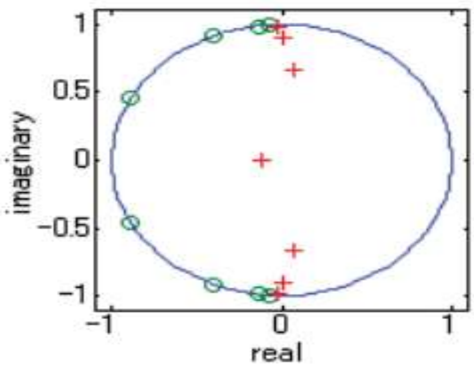
Example 1:$D(\omega) = 1\, for \quad 0 < \omega < 0.2\pi$

& $0\, for\, 0.28\pi < \omega < 1,$

N=7, M=8 and α= 5. The values $\delta_p$and $\delta_s$ of the obtained filter are 0.00357 and 0.0179. The obtained α= $\delta_p/\delta_s$= 5.01




(a)



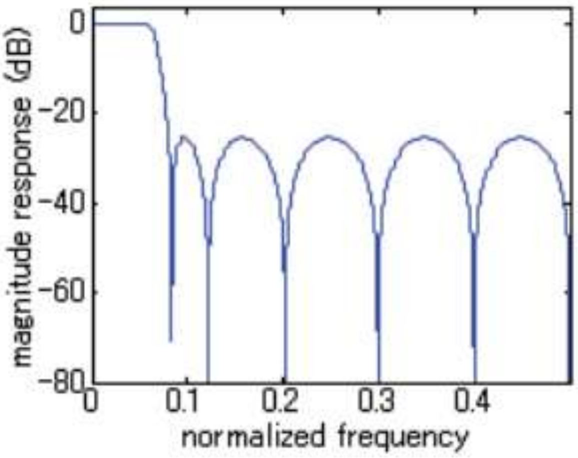

(b)

**Example 1: (a) Magnitude response of H(z);(b) Pole-zero plot**

Example2:$D(\omega) = 1\, for \quad 0 < \omega < 0.12\pi$

& $0\, for\, 0.16\pi < \omega < 1$

N=3, M=11 and α= 3. The values $\delta_p$and $\delta_s$ of the obtained filter are 0.0175 and 0.0526. The obtained α= $\delta_p/\delta_s$= 3.00

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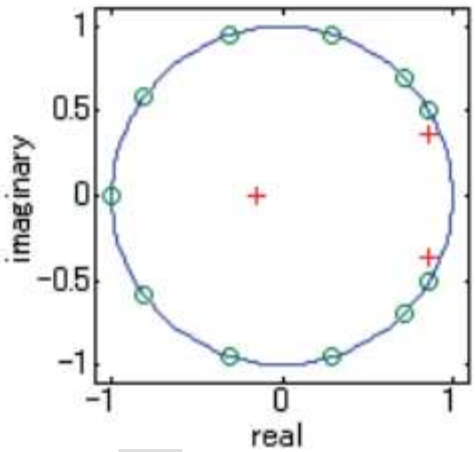


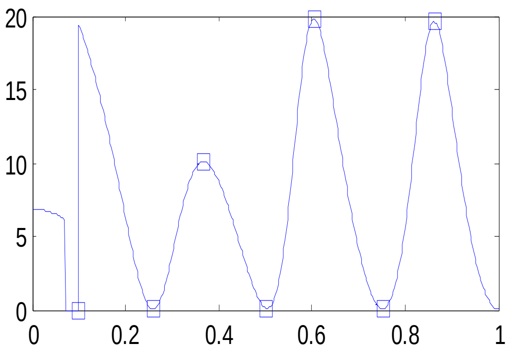

(b)

**Example 2: (a) Magnitude response of H(z);(b) Pole-zero plot**

Example 3: $D(\omega) = 1 \text{ for } 0 < \omega < 0.22\pi$

$\& \quad 0 \; for \; 0.31\pi < \omega < 1$

N=2, M=8 and α= 3. The values $\delta_p$and $\delta_s$ of the obtained filter are 0.0175 and 0.0526. The obtained α= $\delta_p/\delta_s$= 3.00

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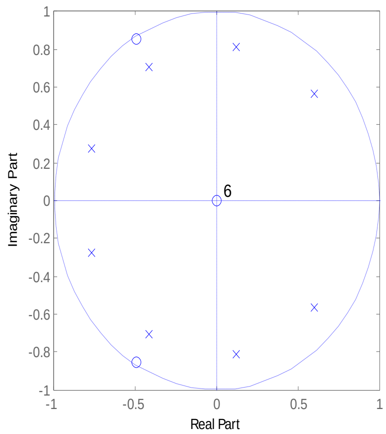


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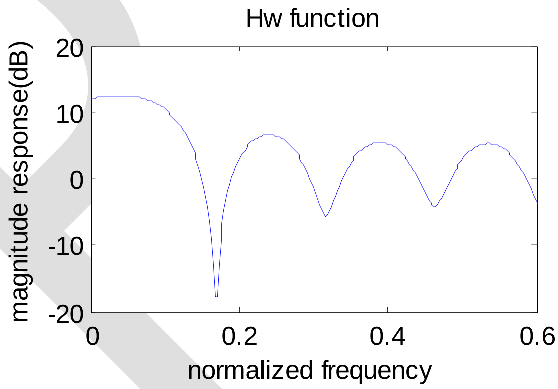


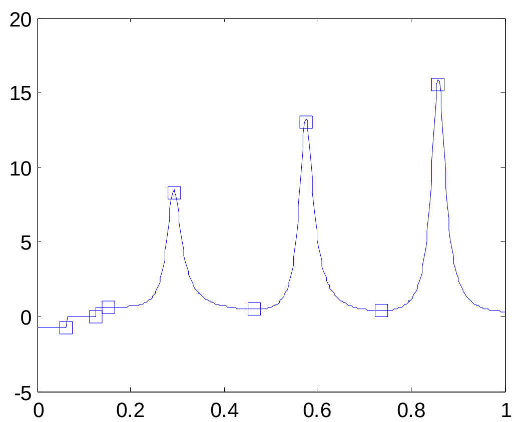

(c)

**Example 3: (a) Local extremal points; (b) Pole-zero plot; (c) Magnitude response of H(z)**

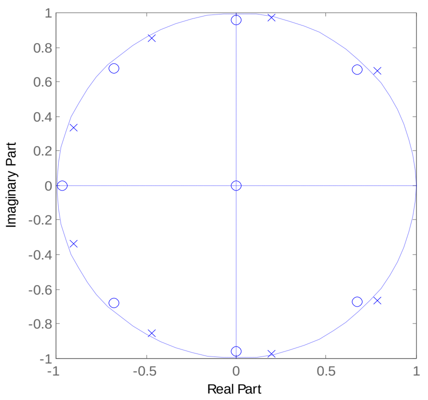
Example 4: $D(\omega) = 1 \text{ for } 0 < \omega < 0.5\pi$

$\& \quad 0 \; for \; 0.52\pi < \omega < 1$

N=7, M=8 and α= 5. The values $\delta_p$and $\delta_s$ of the obtained filter are 0.00357 and 0.0179. The obtained α= $\delta_p/\delta_s$= 5.01




(a)

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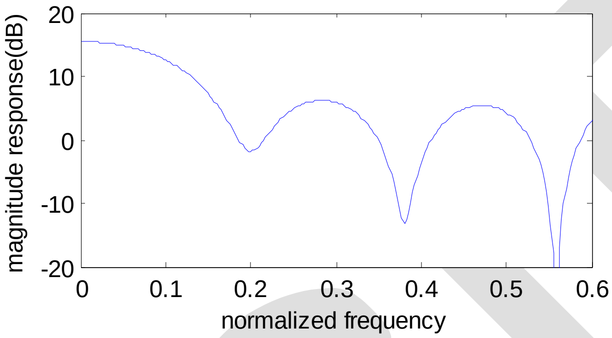


**(c)**

**Example 4: (a) Local extremal points; (b) Pole-zero plot; (c) Magnitude response of H(z)**

## 4. CONCLUSION

In this thesis, design methods of IIR digital filters in the complex domain by transforming the desired frequency response and Complex Chebyshev Approximation to obtain equiripple characteristics are shown. The method used by Complex Chebyshev Approximation is based on the formulation of a generalized eigenvalue problem by using the Rational Remez Exchange Algorithm. Here, the filter coefficients are obtained by solving the eigenvalue problem to find the absolute minimum eigenvalue, and then Complex Chebyshev Approximation is attained through few iterations starting from an initial guess. The proposed algorithm is computationally efficient because it not only retains the speed inherent in the Remez Exchange Algorithm but also simplifies the interpolation step. With this method, IIR digital filter can be easily defined with little iteration and without a special optimization algorithm.

The convergence to the optimal solution is not guaranteed as well as for the conventional rational Remez algorithm

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