



Rearranging the pixel using Log-Cumulant Parameter Estimator and Healing of Restoration Technique

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Abstract

Using second-kind statistics technique noise reduction is not mile stone in the field of image processing, the log-cumulant estimator is proposed for pixel parameter estimation of the log-normal distribution in medical image. The estimates of the shape parameter and scale parameter are independent for the log-cumulant estimator for noise reduction. Parameter estimation results from Monte Carlo simulation demonstrate that, the log-cumulant estimator is insensitive to noise samples and it leads to better performance noise reduction when compared to the moment estimator using Laplacian function.

Keywords:

Healing; Restoration; Parameter estimation; Log-cumulant estimator; Laplacian function.



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Introduction

The log-normal distribution is commonly used for noise reduction model the high resolution sea clutter and the regions of strong spatial variation such as urban areas [1-4]. With two parameters (shape parameter and scale parameter), the log-normal distribution can fit the experimental data better than the one-parameter distributions such as Rayleigh. For example, the Rayleigh distribution usually applies when the radar resolution cell is large so that it contains many scatterers, with no one scattered dominant. However, when the resolution cell size and the grazing angle are small, the log-normal distribution can describe the heavy tails of the clutter more precisely compared to the Rayleigh [2]. In order to use the log-normal distribution in practical applications, its parameters should be estimated accurately. The moment method can be used to estimate the parameters of this distribution [1, 5], but it is sensitive to samples. For the severe impulsive samples, which correspond to the large values of the shape parameter, the performance of the moment estimator is degraded seriously. In this paper, the log-cumulant estimator is proposed for the log-normal distribution based on second-kind statistics, which relies on the Mellin transform [6-8]. We compare the performances of the log-cumulant estimator and the moment estimator, and we have observed that the log-cumulant estimator leads to high estimation accuracy no matter what values are chosen for the shape parameter, which is validated by parameter estimation results from Monte Carlo simulation. Consequently, we recommend the log-cumulant estimator instead of the moment estimator.

Log-Normal Distribution

The log-normal distribution has the following probability density function (pdf) [1, 2]

$$f(x) = \frac{1}{x\sqrt{2\pi V}} \exp\left[-\frac{(\log x - \beta)^2}{2V}\right], x \geq 0$$

Where $V (V > 0)$ is the shape parameter and $(-8 < \beta < +8)$ is the scale parameter. Denoting X as a log-normal random variable with parameters and V, β , its natural logarithm $Y (Y = \log X)$ can be represented by the following pdf

$$f(x) = \frac{1}{\sqrt{2\pi V}} \exp\left[-\frac{(x - \beta)^2}{2V}\right]$$

Obviously Y follows the classical Gaussian distribution with mean β and variance V . For various values of V , the pdf of log-normal distribution is plotted in Figure 1. Obviously, the value of V controls the shape of the pdf. The larger the value of V is the severer skewness the distribution has.

Since the natural logarithm of the log-normal distribution is the Gaussian distribution, the log-normal distribution can be simulated by

$$X = \exp(\sqrt{VZ} + \beta)$$

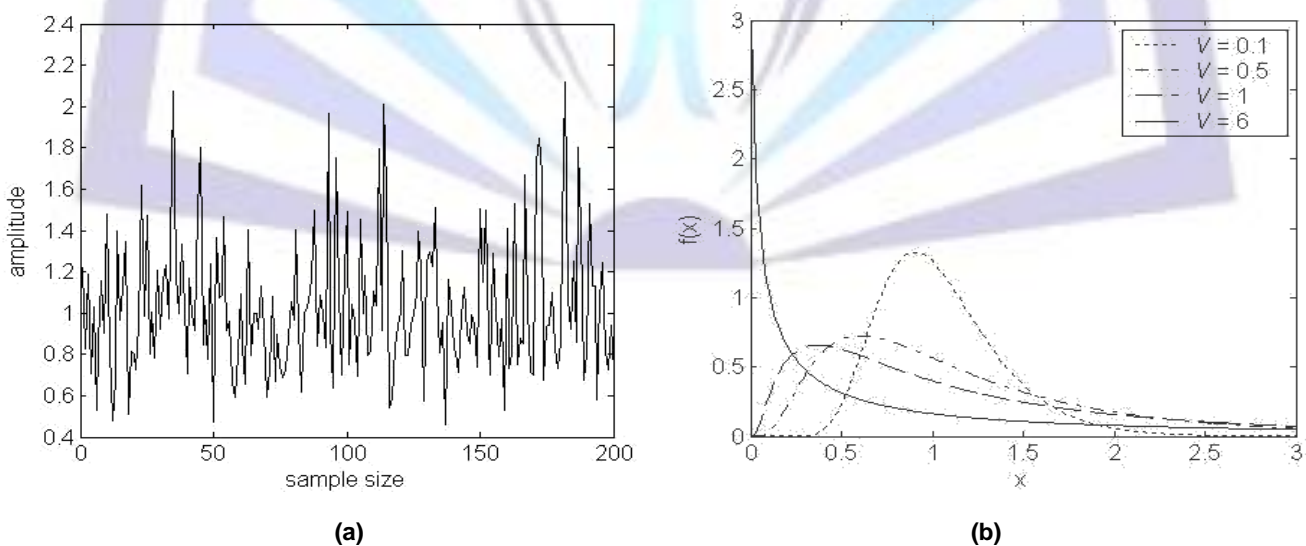


Figure 1. Log-normally distributed samples ($\beta = 0$ and the number of samples is 200)

Figure 2(b). Pdf of log-normal distribution ($\beta = 0$),



Where X is the log-normal random variable with parameters V and β , and Z is the standardized Gaussian random variable with mean 0 and variance 1. With the help of (3), the log-normally distributed samples can be simulated, which are shown in Fig. 2 for various values of V . It is apparent that the log-normal samples with $V = 6$ show much severer impulsiveness than the ones with $V = 0.1$. In general, the larger the value of V is, the more impulsive the log-normal samples are. Since the log-normally distributed samples can be simulated readily, we can use the Monte Carlo simulation to compare the performance of various parameter estimators.

Moment Estimator

The n^{th} order moment of log-normal distribution can be written as n

$$E(X^n) = \exp(n\beta + n^2V / 2), n = 1, 2, \dots$$

Where X is the log-normal random variable with parameters V and β hence, the moment estimator for the log-normal distribution is straightforward as follows [1]:

$$\frac{E(X^2)}{E^2(X)} = \exp(V), E(X) = \exp(\beta + V / 2)$$

By replacing the actual moments with the sample moments, the shape parameter V can be estimated from (5), and then the scale parameter β can be estimated from (6). It should be noted that the estimate of β relies on the estimated values of V , so the error of estimate of V may degrade the estimation accuracy of β

The moment estimator was tested for various true values of parameter V according to Monte Carlo simulation. The log-normally distributed samples were simulated independently by using (3), and the number of samples is 10000. For each V true parameter, the Monte Carlo simulation experiment was repeated 100 times independently, and then the average and standard deviation of the estimates was V computed. The results are shown in Table 1 with standard deviations in parentheses. Obviously, the performance of the moment estimator relies on the true values of β . For the smaller values of V , the moment estimator can lead to high estimation accuracy. However, if the larger values are chosen for the (e.g., $V = 0.1$ $V = 6$), the moment estimator results in poor performance. In other words, the moment estimator is sensitive to samples. If the samples show severe impulsiveness (e.g. $V = 6$), the moment estimator cannot achieve high estimation accuracy.

True Value	V=0.1	V=0.5	V=1	V=2	V=5	V=6
V	0.0998 (0.0017)	0.4999 (0.0138)	0.9971 (0.0467)	1.9385 (0.2015)	4.1894 (0.5715)	4.6268 (0.6755)
β	0.0003 (0.0032)	0.0010 (0.0089)	0.0022 (0.0218)	0.0303 (0.0916)	0.3994 (0.2332)	0.6625 (0.2449)

Table1. Monte Carlo Simulation of Moment Estimator (true $\beta = 0$)

Log-Cumulant Estimator

The log-cumulant estimator is based on second-kind statistics, which relies on the Mellin transform [6-8]. Denoting g as a function defined over $[0, +\infty)$ its Mellin transform is defined as

$$M[g(x)](s) = \int_0^{+\infty} x^{s-1} g(x) dx, M[g(y)](s) = \int_0^{+\infty} y^{s-1} g(y) dy$$

By the above equations using Laplacian function/theorem

$$M[g(xy)](s) = \int_0^{+\infty} x^{s-1} g(x) dx + \int_0^{+\infty} y^{s-1} g(y) dy$$

where s is the complex variable of the Laplacian Transform. Specifically, for a pdf f defined in $[0, +\infty)$ the second-kind first characteristic function is defined by $\phi(s) = \int_0^{+\infty} x^{s-1} f(x) dx$ Then, the second-kind second characteristic



function is defined by $\varphi(s) = \log(\phi(s))$, Finally, the nth order second-kind cumulant (log-cumulant)

is defined by $k_r = \frac{d^r \varphi(s)}{ds^r} \Big|_{s=1}$, The first two log-cumulants k_1 and k_2 can be estimated empirically from N samples Y_i as follows:

$$k_1 = \frac{1}{N} \sum_{i=1}^N [\log(y_i)], \quad k_2 = \frac{1}{N} \sum_{i=1}^N [(\log(y_i) - k_1)^2]$$

By substituting (1) into (8) and after some manipulation, the second-kind first characteristic function of log-normal distribution is given by

$$\phi(s) = \exp[\beta(s-1) + \frac{(s-1)^2}{2}]$$

Then, substituting (13) into (9) and subsequently into (10), the log-cumulant estimator (i.e., the first and second orders log-cumulants) for the log-normal distribution is finally obtained as follows:

$$K_1 = \beta, \quad K_2 = V$$

By replacing the actual log-cumulants with the sample log-cumulants (equations (11) and (12)), parameters β and V can be easily estimated from (14) and (15), respectively. Compared to the moment estimator (equations (5) and (6)), the estimate of the scale parameter β in the log-cumulant estimator is independent with that of the shape parameter V .

The log-cumulant estimator was tested on Monte Carlo simulations for various true values of parameter V . For each true parameter V , the Monte Carlo simulation V experiment was repeated 100 times independently, and the number of samples was 10000 for each time. Table 2 illustrates the average and standard deviation values (in parentheses) of Monte Carlo simulation results based on the log-cumulant estimator, and Fig. 3 shows the performance comparison of the log-cumulant estimator and the moment estimator as a function of true V . Obviously, the log-cumulant estimator leads to high estimation accuracy no matter what values are chosen for the true V . Therefore, the log-cumulant estimator is robust and not sensitive to samples.

True Value	V=0.1	V=0.5	V=1	V=2	V=5	V=6
V	0.1000 (0.0015)	0.5000 (0.0061)	1.0006 (0.0143)	2.0001 (0.0244)	5.0001 (0.0793)	6.0006 (0.0793)
β	0.0002 (0.0028)	0.0003 (0.0068)	0.0010 (0.0089)	0.0007 (0.0136)	0.0010 (0.0221)	0.0013 (0.0238)

Table2. Monte Carlo Simulation of Log-Cumulant Estimator (true $\beta = 0$)

Even for the samples with severe impulsiveness (e.g., $V = 6$), the log-cumulant estimator can achieve high estimation accuracy. For the moment estimator, on the other hand, the estimated parameters may be close to that of the log-cumulant estimator for the smaller values of V (e.g., $V = 0.1$). However, as shown in Table 1, the performance of the moment estimator is deteriorated for the larger values of (e.g. $V=6$). In a word, the log-cumulant estimator is superior to the moment estimator, which is validated by the Monte Carlo simulations.

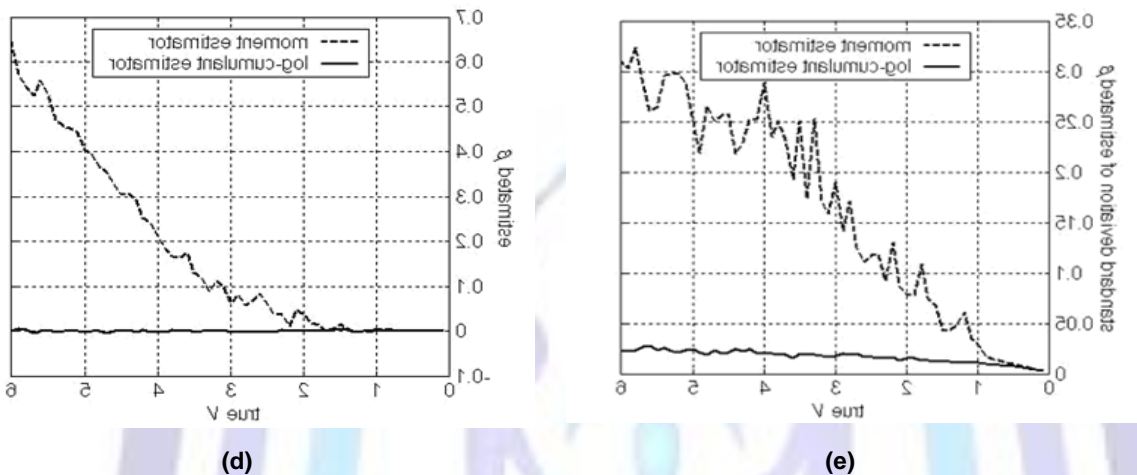
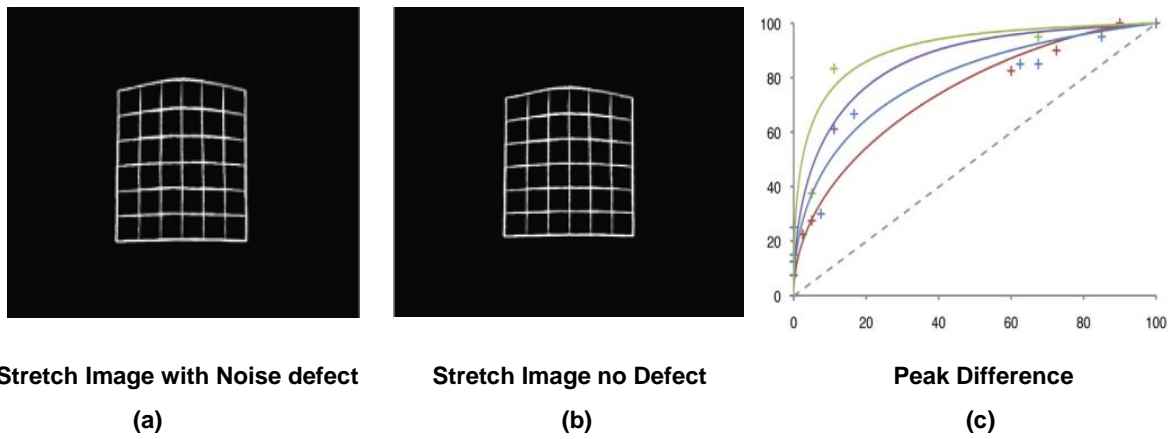


Figure3: (a, b, c, d, e) Log-Cumulant noise reduction using Laplacian

Conclusion

The log-cumulant estimator based on the second-kind statistics using Laplacian function is proposed to estimate the noise reduction parameters of log-normal distribution in the paper. Compared to the moment noise estimator, the noise estimates of the shape parameter and scale parameter in the log-cumulant estimator are independent with each other, and the log-cumulant estimator leads to high estimation accuracy no matter what values are chosen for the shape parameter, which is demonstrated by parameter estimation results from Monte Carlo simulations. The log-cumulant noise reduction estimator is insensitive to samples, and it can achieve good performance even for the severely noise impulsive samples. Therefore, the log-cumulant estimator is recommended instead of the moment noise estimator.

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