Soft Compactness and Soft Topological Separate axioms

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Abstract

In this paper, we further discuss soft compactness and soft separation axioms in the soft topological space over the rough soft formal context $T=(G,M,R,F)$. We define the compact soft topological space , give soft separation axioms of soft rough topological space and study their relationship in the soft topological space over the rough soft formal context.

Keywords

rough soft formal context, soft rough topological space, soft open(closed) set, countable(finite) cover, compactness.

1 Introduction

Topology $Euler, M$ can be formally defined as the study of qualitative properties of certain objects (called topological spaces) that are invariant under a certain kind of transformation (called a continuous map). Topology has been become one of the great unifying ideas of mathematics, and topology can be applied to many field, for example, when the fundamental concept of a fuzzy set was introduced by Zadeh in 1965 L , Chang C introduced fuzzy topological spaces in 1968. Later on, Many researchers studied and developed it, such as, author replaced the closed interval by a more general lattice A . Ying $ying$ studied Hohles topology and called it fuzzifying topology.

The uncertainty of data modeling the problems in engineering, physics, computer sciences, economics, social sciences, medical science and many other diverse fields. And formal concept context $Wille, ganter$, rough set $Pawlak, z, kent$, soft set $molodsov, p$ are extensively applied in the uncertainty reasoning.

Shabir and Naz $Shbir$ launched the study of soft topological spaces which was defined over an initial universe with a fixed set of parameters, and gave the concepts including soft open(closed)set, soft interior points and so on, they defined and discussed the soft T_i-space .Authors $Naim$ defined soft topology on a soft set, and presented its related properties. Authors $Sabir$ continued investigating the properties of soft topological spaces. Some authors zh discussed the relationship among soft sets, soft rough sets and topologies.

Author li defined the rough soft formal context based on rough formal context fu , and discussed the rough properties of rough formal context in soft set. Author fl defined some topological operators in the rough soft formal context, defined the soft rough topological space, and some topological properties were discussed over the soft rough topological space. Authors fli discussed the soft topological separation axioms in the rough soft formal context.

In this paper, we continue to study the soft rough topology, and we discuss soft compactness and soft separation axioms in the soft topological space over the rough soft formal context . The rest of this paper is organized as following. In section 2, we review some basic concepts and properties of rough concept formal soft sets, and soft rough topology , soft rough topological space over the rough soft formal context $T=(G,M,R,F)$. In section 3, we define the compact soft topological space , give soft separation axioms of soft rough topological space and study the relationship between compactness and separation axioms in the soft topological space over the rough soft formal context $T=(G,M,R,F)$. Conclusions are given in section 4.

2 Basic knowledge

Definition 2.1 M Let $\overset{\circ}{A}$ be a collection of sets, be a set, if $B\subseteq\bigcup_{A\in A}\overset{\circ}{A}$, then $\overset{\circ}{A}$ is a **cover** of B , if $\overset{\circ}{A}$ is a countable class or a finite class , then $\overset{\circ}{A}$ is a **countable cover** or **a finite cover of** B ; if $\overset{\circ}{A_1}\subseteq\overset{\circ}{A}$, and $\overset{\circ}{A_1}$ is a cover of B , then $\overset{\circ}{A_1}$ is a sub-cover of $\overset{\circ}{A}$ for B .

Definition 2.2 $molodsov$ Let U be an initial universe set and E be a set of parameters. Let $\wp(U)$ denotes the power set of U and $A \subset E$. Then a pair (F, A) is called a **soft set** over U, where $F : A \to \wp(U)$ is a mapping.

That is, the soft set is a parameterized family of subsets of the set U. Every set $F(e)$, $\forall e \in E$, from this family may be considered as the set of *e*-**elements** of the soft set (F, E), or considered as the set of *e*-**approximate** elements of the soft set.

Definition 2.3 li Let (G, M, R) is a rough formal context, G is objects set, is also called the universe, M is attributes set. A pair (F, B) is a soft set over G, where and $F : B \to \wp(U)$ is a set-value mapping over G, furthermore, the lower and upper rough approximations of pair (F, B) are denoted by

$$\underline{R}(F, B) = (\overline{F}, B),\ \overline{R}(F, B) = (\overline{F}, B),$$

which are soft sets over G with the set-valued mappings given by $\underline{F}(x) = \underline{B}(F(x))$ and $\overline{F}(x) = \overline{B}(F(x))$, where $x \in B$. The operators $\overline{R}, \underline{R}$ are called the **lower and upper rough approximation operators** on soft set (F, B).

If $\overline{R} = \underline{R}$, we say that the soft set (F, B) is **definable**, otherwise, (F, B) is **rough**.

we call such quadruple tuple (G, M, R, F) as **rough soft formal context**, and, such soft set (F, B) on the rough soft formal context (G, M, R, F) which is called **rough soft formal set**.

Obviously, $\forall x \in B \subseteq M, F(x) \subseteq G$ is a parameterized family of subsets of G, and $F(x)$ is the set of x - approximate elements in (G, M, R, F).

Definition 2.4 li Let (G, M, R, F) be a rough soft formal context with the objects set G, and attributes set M. $B_1, B_2 \subseteq M$, (F, B_1) and (F_1, B_2) are two soft sets over G on the rough soft formal context (G, M, R, F). $F_1 : B \to \wp(G)$ is a set-value mapping over the rough soft formal context (G, M, R, F).

(i) If $B_1 \subseteq B_2$, and $\forall x \in B_1 \subseteq B_2$, having $F(x) \subseteq F_1(x)$, then the soft sets (F, B_1) is a **soft subset** of the soft set (F_1, B_2), denoted as $(F, B_1) \widetilde{\subset} (F_1, B_2)$.

(ii) Two soft sets (F, B_1) and (F_1, B_2) on the rough soft formal context (G, M, R, F) are said **soft equal**, if $(F, B_1) \widetilde{\subset} (F_1, B_2)$, and $(F_1, B_1) \widetilde{\subset} (F, B_2)$. We simply denote by $(F, B_1) = (F_1, B_2)$.

(iii) The **relative complement** of (F, B) is denoted by $(F, B)^c$ and is defined by $(F, B)^c = (F^c, B)$, where $F^c : B \to \wp(G)$, and $F^c(x) = G - F(x), \forall x \in B$.

Clearly, $((F, B)^c)^c = (F, B)$.

(iv) (F, B) is said to be a **relative null rough soft formal set** denoted by $\mathcal{N}$, if $\forall x \in B, F(x) = \varnothing$; if $B = M$, then is called **absolute null rough soft formal set**, denoted as $\widetilde{\varnothing}$.

(v) (F, B) is said to be a **relative whole rough soft formal set** denoted by $\widetilde{G}$, if $\forall x \in B, F(x) = G$.

Definition 2.5 li Let (G, M, R, F) be the rough soft formal context, (F_1, B_1) and (F_2, B_2) are two rough soft formal sets over (G, M, R, F), in which $F_1, F_2 : B \to \wp(G)$ are two set-value mappings.

(i) The **union** (F_1, B_1) and (F_2, B_2) is the rough soft formal set (H, C), where $C = B_1 \bigcup B_2$, and $\forall e \in C$, denoted as

$(F_1, B_1) \widetilde{\bigcup} (F_2, B_2) = (H, C) = (H, B_1 \bigcup B_2)$ where

(ii) The **intersection** of (F_1, B_1) and (F_2, B_2) is the soft rough formal set (H, C) is denoted as $(F_1, B_1) \bigcap (F_2, B_2)$ and is defined as $(F_1, B_1) \sqcap (F_2, B_2) = (H, C)$, where $C = B_1 \bigcap B_2$, and $\forall e \in C$, $H(e) = F_1(e) \bigcap F_2(e)$.

Definition 2.6 fl Let $T = (G, M, R, F)$ be a rough soft formal context over the object set G and attributes set $M, B_i \subseteq M$ which is the collection of soft sets on the rough soft formal context (G, M, R, F), if it is satisfied:

(1) $\widetilde{\emptyset} \in \tau$, $\widetilde{G} \in \tau$

(2) The union of any number of soft sets in τ belongs to τ, that is, τ is closed for the any union of soft sets over T.

(3) The intersection of any two soft sets in τ belongs to τ, that is, τ is closed for the finite intersection of soft sets over T.

Then the collection τ is called **a soft topology** over the rough soft formal context T (simply called **soft rough topology**).

The triplet (G, τ, M) is called **a soft topological space** over the rough soft formal context T (simply called **soft rough topological space**).

The members of τ are soft open sets in T, the relative complement $(F, B)^c = (F^c, B)$ is said to be a **soft closed** set in T if $(F, B)^c \in \tau$.

3   The soft compact topological space and soft topological separate axioms

In the following, we study based on the rough soft formal context $T = (G, M, R, F)$ in which G is an initial universe set M is a set of attributes, (G, τ, M) be a soft topological space over T set, and (F, B) is a soft set over T.

Definition 3.1   Let (G, τ, M) be a soft topological space over T, (F, B) be a soft set over G, and $x \in G$. we say $x \in (F, B)$ and read as x **belongs to** the soft set (F, B), if $x \in F(e)$ for all $e \in B$ and if there is some $e \in B$, such that $x \notin F(e)$, then $x \notin (F, B)$, read as x does not belong to the soft set (F, B).

Definition 3.2   Let (G, τ, M) be a soft topological space over T, G is an initial universe set, M is a set of attributes, and (F, B) is a soft set over G, $x \in G$, then:

(i) **soft interior** of soft set (F, B) over G is denoted by $(F, B)^{\circ}$ and is defined as the union of all soft open set contained in (F, B), that is, $(F, B)^{\circ} = \widetilde{\bigcup} \{(F_i, B_i) \widetilde{\subseteq} (F, B) \in \tau\}$. Such $(F, B)^{\circ}$ is the largest soft open set contained in of (F, B);

(ii) The **soft closure** of (F, B), denoted by $\overline{(F, B)}$ is the intersection of all soft closed super set of soft set (F, B), that is $\overline{(F, B)} = \bigcap \{(F_i, B_i) \mid (F, B) \subseteq (F_i, B_i) \in \tau\}$. Clearly, $\overline{(F, B)}$ is the smallest soft closed set over T which contain the soft set (F, B);

(iii) If there is a soft open set (F_1, B_1) over G, such that $x \in (F_1, B_1) \widetilde{\subseteq} (F, B)$, then x is a **soft interior point** of (F, B) and (F, B) is the **soft neighborhood** of x.

Similarly, (F_1, B_1); (F, B) are two soft sets over G, if $(F_1, B_1) \widetilde{\subseteq} (F, B)^{\circ}$, then (F, B) is the **soft**

neighborhood of (F_1, B_1) .

Specially, if (F, B) is a soft open (closed) set, then (F, B) is a soft open (closed) neighborhood of x .

Definition 3.3 Let (G, τ, M) be a soft topological space over T , (F, B) be a soft set over G , and the collection of

soft sets $\overset{\circ}{A} = \{(F, B) \,|\, (F, B) \text{ is soft open (closed)}, (F, B) \in \tau\}$ is a cover of G , then A is a soft **open(closed)**

cover of G .

Definition 3.4 Let (G, τ, M) be a soft topological space over G , is the collection soft open sets over G , and

$x, y \in G$ and $x \neq y$, if there exist soft neighborhood (F_1, M) of x and soft neighborhood (F_2, M) of y such that

$y \notin (F_1, M)$ or $x \notin (F_2, M)$, that is , for any $x \neq y \in G$, there exist the soft open sets $(F_1, M), (F_2, M) \in \tau$, such

that either

$$\text{(i) } x \in (F_1, M) \text{ and } y \notin (F_1, M) \quad \text{or}$$

$$\text{(ii) } y \in (F_2, M) \text{ and } x \notin (F_2, M),$$

then soft rough topological space (G, τ, M) over G is called a **soft rough** T_0-**space**.

Obviously, if (G, τ, M) is a soft rough topological space, in which τ is a trivial soft rough topology over G , and

$|G| \geq 2, |G|$ represents the cardinality of G , that is , the numbers of G . Then (G, τ, M) over G is not a soft rough

T_0-space.

Definition 3.5 Let (G, τ, M) be a soft topological space over G , is the collection soft open sets over G , and

$x, y \in G$ and $x \neq y$, if there exist soft neighborhood (F_1, M) of x and soft neighborhood (F_2, M) of y such that

$y \notin (F_1, M)$, that is , for any $x \neq y \in G$, there exist the soft open sets (F_1, M) , $(F_2, M) \in \tau$, such that

$x \in (F_1, M), y \notin (F_1, M)$ and $y \in (F_2, M), x \notin (F_2, M)$, then soft topological space (G, τ, M) over G is called a

soft rough T_1 -**space**.

Obviously, if the triplet (G, τ, M) is a soft rough T_1 -space, then it is also a T_0 -space, however, the converse does

not hold.

Definition 3. 6 Let (G, τ, M) be a soft topological space over G , is the collection soft open sets over G , and

$x, y \in G$ and $x \neq y$, if there exist soft neighborhood (F_1, M) of x and soft neighborhood (F_2, M) of y such that

$y \notin (F_1, M), x \notin (F_2, M)$, and $(F_1, B_1) \sqcap (F_2, B_2) = \widetilde{\emptyset}$, then soft topological space (G, τ, M) over G is called a

soft rough T_2 -**space**, which is also called a soft **rough Hausdröff space**.

Definition 3.7 Let the triplet (G, τ, M) be a soft topological space over G , for all $x \in G$ and (F, M) is a soft

closed set in T , if there exist two soft open sets $(F_1, M), (F_2, M)$ in τ , such that $x \in (F_1, M)$, and

$(F, M) \widetilde{\subseteq} (F_2, M)$, and $(F_1, M) \sqcap (F_2, M) = \widetilde{\emptyset}$, then soft topological space (G, τ, M) is called a soft rough **regular**

space.

Definition 3.8 Let the triplet (G, τ, M) be a soft topological space over G , (G, τ, M) be a soft rough

topological space over G , (F_1, M) , (F_2, M) are soft closed sets in τ , if there exist soft open sets (F_3, M) , (F_4, M) in

τ such that $(F_1, M) \widetilde{\subseteq} (F_3, M)$, $(F_2, M) \widetilde{\subseteq} (F_4, M)$, and $(F_3, M) \sqcap (F_4, M) = \widetilde{\emptyset}$, then soft topological space

(G, τ, M) is called a **soft rough normal space**.

Definition 3.9 Let (G, τ, M) be a soft topological space over T, (F, B) be a soft set over T, and the collection of

soft sets $\overset{\circ}{A} = \{(F, B) \,|\, (F, B) \text{ is an soft open set in } \tau\}$ be any soft open cover of G, if A has a soft countable

subcover of G, then (G, τ, M) is a **soft Lindelöff topological space** over T.

Obviously, let (G, τ, M) be a discrete soft topological space over T, and (F, B) be a soft set over G, suppose

that the soft topology $\tau = \{(F_i, M) \,|\, F_i : M \to \wp(G)\}$ is a discrete soft rough topology over T. Then (G, τ, M) is not a

soft Lindel$\ddot{o}$ff topological space over T.

Definition 3.10 Let (G, τ, M) be a soft topological space over T, (F, B) is a soft set over T, and the collection of soft

sets $\overset{\circ}{A} = \{(F, B) \,|\, (F, B) \text{ is an soft open set in } \tau\}$ be a soft open cover of G, if any soft open cover of G has a soft

finite sub-cover, then (G, τ, M) is a compact soft topological space over T.

Clearly, if (G, τ, M) is a compact soft topological space over T, then it must be a Lindelöff soft topological space over T,

however, the converse does not hold.

Proposition 3.1 Let (G, τ, M) be a soft topological space over T, if (F, B) is any finite soft subset of $\tilde{G}$, then

(F, B) is a compact soft subset of $\tilde{G}$.

Proof Let (G, τ, M) be a soft topological space over T, and (F, B) be any finite soft subset of $\tilde{G}$, the collection

$\overset{\circ}{A} = \{(F, B) \,|\, (F, B) \text{ is soft set in } T, (F, B) \widetilde{\subseteq} \tilde{G}\}$ covers (F, B), for all $x \in (F, B)$, that is, $x \in F(e)$ for all

$e \in B$, taking $\overset{\circ}{A_1} = \{(F, B) \,|\, (F, B) \text{ is soft set in } T, \text{and } x \in (F, B)\}$, then $\overset{\circ}{A_1}$ is a finite sub-cover of (F, B).

Hence, (F, B) is a compact soft subset of $\tilde{G}$.

Proposition 3.2 Let $(G, \tau_1, M), (G, \tau_2, M)$ be two soft topological spaces over T, $\tau_1 \subseteq \tau_2$, and (G, τ_2, M)

be a compact soft topological spaces over T, then (G, τ_1, M) is also a compact soft topological spaces over T.

Proof: Let $(G, \tau_1, M), (G, \tau_2, M)$ be two soft topological spaces over T. Suppose the collection

$\overset{\circ}{A} = \{(F, B) \,|\, (F, B) \text{ is a soft set in } T, (F, B) \widetilde{\subseteq} \tilde{G}\}$ which is an open cover of τ_1, by $\tau_1 \subseteq \tau_2$ having the

collection $\overset{\circ}{A}$ is also an open cover of τ_2, and (G, τ_2, M) be a compact soft topological spaces over T, then there exist

a finite sub-cover $\overset{\circ}{A_1}$ of $\overset{\circ}{A}$ which is a finite open cover of τ_1.

Hence, (G, τ_1, M) is also a compact soft topological spaces over T.

Proposition 3.3 Let (G, τ, M) be a soft topological space over T if $(F_1, B_1), (F_2, B_2), \cdots, (F_n, F_n)$ are compact soft

subsets of G, then $(F_1, B_1) \widetilde{\cup} (F_2, B_2) \widetilde{\cup} \cdots \widetilde{\cup} (F_n, F_n)$ is a compact soft subset of G.

Proof: Let (G, τ, M) be a soft topological space over T and $(F_1, B_1), (F_2, B_2), \cdots, (F_n, B_n)$ be compact soft

subsets of $\tilde{G}$. Assume $U = \bigcup_{i=1}^{n} (F_i, B_i)$ and the collection $\overset{\circ}{A} = \{(F, B) \,|\, (F, B) \text{ is a soft set in } T, (F, B) \widetilde{\subseteq} \tilde{G}\}$ which is

any open cover of U , then the collection is also an open cover of $(F_i, B_i), i = 1, 2, \cdots, n,$ by (F_i, B_i) being a compact soft subset of $\tilde{G}$, so there exists a sub-collection $\overset{\circ}{A_i}$ of $\overset{\circ}{A}$ which is sub-cover of $(F_i, B_i), i = 1, 2, \cdots, n,$ and $\bigcap_{i=1}^{n} \overset{\circ}{A_i}$ is a finite cover of U . Hence, $U = \bigcup_{i=1}^{n} (F_i, B_i)$ is a compact soft subset of $\tilde{G}$.

Similarly, we have the following property:

Proposition 3.4 Let (G, τ, M) be a soft topological space over T , the collection

$\overset{\circ}{A} = \{(F, B) | (F, B) \text{ is a compact soft closed subset of } \tilde{G}\},$ then $\bigcap_{(F,B) \in \overset{\circ}{A}} (F, B)$ is a compact soft subset of $\tilde{G}$.

Proof Let (G, τ, M) be a soft topological space over T , and

$\overset{\circ}{A} = \{(F, B) | (F, B) \text{ is a compact soft closed subset of } \tilde{G}\},$ then $\sqcap_{(F,B) \in \overset{\circ}{A}} (F, B)$ is a soft closed subset of $\tilde{G}$ .and

$\forall (F, B) \in T, \sqcap_{(F,B) \in \overset{\circ}{A}} (F, B) \tilde{\subset} (F, B)$. Hence, $\sqcap_{(F,B) \in \overset{\circ}{A}} (F, B)$ is a compact soft subset of $\tilde{G}$.

Furthermore, in soft Hausdröff space, we have:

Furthermore, in soft Hausdr$\ddot{o}$ff space, we have:

Proposition 3.5 Let (G, τ, M) be a soft Hausdröff space over T , the collection

$\overset{\circ}{A} = \{(F, B) | (F, B) \text{ is a compact soft closed subset of } \tilde{G}\},$ then $\sqcap_{(F,B) \in \overset{\circ}{A}} (F, B)$ is a compact soft subset of $\tilde{G}$.

Proof: Let (G, τ, M) be a soft Hausdröff space over T , and

$\overset{\circ}{A} = \{(F, B) | (F, B) \text{ is a compact soft closed subset of } \tilde{G}\},$ then $\sqcap_{(F,B) \in \overset{\circ}{A}} (F, B)$ is a soft closed subset of $\tilde{G}$ .and

$\forall (F, B) \in T, \sqcap_{(F,B) \in \overset{\circ}{A}} (F, B) \tilde{\subset} (F, B)$. Hence, $\sqcap_{(F,B) \in \overset{\circ}{A}} (F, B)$ is a compact soft subset of $\tilde{G}$.

Theorem 3.1 Let (G, τ, M) be a soft Hausdröff space over T , the collection

$\overset{\circ}{A} = \{(F, B) | (F, B) \text{ is a compact soft closed subset of } \tilde{G}\},$ then $\sqcap_{(F,B) \in \overset{\circ}{A}} (F, B)$ is a compact soft subset of $\tilde{G}$.

Proof： Suppose that A is a compact subset of $\tilde{G}$, and $x \notin A, \forall y \in A,$ by (G, τ, M) being a soft Hausdröff space over T , then there are open neighborhoods U_x of x and V_x of y, such that $U_x \sqcap V_x = \tilde{\emptyset}$.

The collection $\{V_y | y \in A\}$ is an open cover of compact subset A, which has a finite sub-collection, assume that

$\{V_{y_1}, V_{y_2}, \cdots V_{y_n}\}$ covers A, and let $U = \bigcap_{i=1}^{n} U_{y_i}, V = \bigcap_{i=1}^{n} V_{y_i}$ be the open neighborhood of x and A, respectively, and for all

$i = 1, 2, \cdots, n$, having $U \sqcap V_{y_i} = U_{y_1} \sqcap U_{y_2} \sqcap \cdots U_{y_n} \sqcap V_{y_i} = \tilde{\emptyset}$.

So, $U \sqcap V = (U \sqcap V_{y_1}) \tilde{\cup} (U \sqcap V_{y_2}) \tilde{\cup} \cdots \tilde{\cup} (U \sqcap V_{y_n}) = \tilde{\emptyset}$.

Corollary 3.1 Every compact subset of soft Hausdröff space over T is a soft closed set.

Proof Let (G,τ,M) be a soft Hausdröff space over T , and (F,B) is a compact soft subset of $\tilde{G}$ 。

$\forall x \in G, x \notin (F,B)$, then $x \notin U \sqcap (\tilde{G}-\{x\})$, in which , U is any open soft neighborhood of (F,B) , so, (F,B) is a soft closed set.

Corollary 3.2 Let (G,τ,M) be a soft soft Hausdröff space over T , and (F,B) be a soft closed set if and only if (F,B) is a compact subset of $\tilde{G}$.

We have

$$Compact\ soft\ topological\ space:\quad closed\ soft\ set \Rightarrow compact\ soft\ subset$$
$$soft\ Hausdorff\ space:\quad\quad\quad closed\ soft\ set \Leftarrow compact\ soft\ subset$$
$$Compact\ soft\ Hausdorff\ space:\quad closed\ soft\ set \Leftrightarrow compact\ soft\ subset$$

Corollary 3.3 Every compact   soft   Hausdröff   space over T is a regular soft topological space.

Proof Let (G,τ,M) be soft Hausdröff space over T , and (F,B) be a soft closed set over (G,τ,M). For all $x \in G, x \notin (F,B)$, then (F,B) is a compact soft subset, and there exist open neighborhoods U ans V of x and (F,B), respectively, such that $U \sqcap V = \varnothing$, that is , (G,τ,M) is a regular soft topological space over T .

Theorem 3.2 Let (G,τ,M) be a soft Hausdröff space over T , and $(F_1,B_1),(F_2,B_2)$ be compact subsets of $\tilde{G}$, and $(F_1,B_1) \sqcap (F_2,B_2) = \tilde{\varnothing}$, then there exist soft open neighbors U and V of (F_1,B_1) and (F_2,B_2), such that $U \sqcap V = \tilde{\varnothing}$.

Proof Suppose that (F_1,B_1),(F_2,B_2) are compact subsets of $\tilde{G}$, and $(F_1,B_1) \sqcap (F_2,B_2) = \tilde{\varnothing}$. For all

$x \in (F_1,B_1)$, there exists an open soft neighborhood U_x and V_x of x and (F_2,B_2), such that $U_x \sqcap V_x = \tilde{\varnothing}$, and the collection $A = \{U_x | x \in (F_1,B_1), (F_1,B_1) \in \tau\}$ is an open cover of (F_1,B_1) ,then there exists a finite sub-collection $\{U_{x_1}, U_{x_2}, \cdots, U_{x_n}\}$ of U_x which covers (F_1,B_1), similarly, the finite sub-collection $\{V_{x_1}, V_{x_2}, \cdots, V_{x_n}\}$ of V_x which

covers (F_2,B_2), Assume $U = \bigcup_{i=1}^{n} U_{x_i}, V = \bigcup_{i=1}^{n} V_{x_i}$,then $U_{x_i} \sqcap V = \tilde{\varnothing}$, so $U \sqcap V = \tilde{\varnothing}$.

Proposition 3.5 Let (G,τ,M) be a soft regular topological space over T , and (F,B) be a compact subset of $\tilde{G}$,and $(F_1,B_1) \tilde{\subset} \tilde{G}$,If $(F,B) \tilde{\subset} (F_1,B_1) \tilde{\subset} \overline{(F,B)}$,then (F_1,B_1) is a compact subset of $\tilde{G}$.

Proof Suppose that the collection $\overset{\circ}{A}$ of soft set in τ is an open cover of (F_1,B_1), by $(F,B) \tilde{\subset} (F_1,B_1) \tilde{\subset} \overline{(F,B)}$,

and (F,B) be a compact subset of $\tilde{G}$, having the collection A is also an open cover of (F,B) ,so there exists a finite sub-collection A_1 of A which is a finite cover of (F,B) ,and by (G,τ,M) being a soft regular topological space over T ,then there is an open soft neighborhood V of (F,B) ,such that $V \tilde{\subset} \tilde{\bigcup}_{U \in A_1} U$,and $(F_1,B_1) \tilde{\subset} \overline{(F,B)} \tilde{\subset} V \tilde{\subset} \tilde{\bigcup}_{U \in A_1} U$, that is , A_1 is a finite open cover of (F_1,B_1) .Hence, (F_1,B_1) is a compact subset of $\tilde{G}$.

Proposition 3.6 Let (G, τ, M) be a soft regular topological space over T , and (F, B) be a compact subset of $\tilde{G}$. U is an open soft neighborhood of (F, B), then there is an open soft neighborhood V of (F, B) such that $\overline{V} \tilde{\subset} U$.

Proof: Suppose that (F, B) is a compact subset of $\tilde{G}$ over the soft regular topological space (G, τ, M). U is an open soft neighborhood of (F, B), for all $x \in (F, B)$, there is an open neighborhood of V_x of x such that $V_x \tilde{\subset} U$,and the collection $\{V_x \mid x \in (F, B)\}$ is an open cover of (F, B), then there has a finite sub-collection $\{V_{x_1}, V_{x_2}, \cdots, V_{x_n}\}$ of U_x which covers (F, B), Assume $V = \bigcup_{i=1}^{n} V_{x_i}$, then V is an open soft neighborhood of (F, B) , and

$$\overline{V} = \overline{(\bigcup_{i=1}^{n} V_{x_i})} = \bigcup_{i=1}^{n} \overline{V_{x_i}} \tilde{\subset} U .$$

4 Conclusion

In this paper, we define the compact soft topological space , give soft separation axioms of soft rough topological space, we study soft compactness and soft separation axioms in the soft topological space over the rough soft formal context $T = (G, M, R, F)$ and give their properties. That is, we investigate the compactness and separate axioms of topological space combing topological structure and soft sets with rough sets and formal context, some different types of hybrid models are presented, which is topology over the rough soft formal context, and we name it as the soft rough topology. That offers a new method and tool in data analysis. We also can discuss more topological properties, such as countable axioms over the rough soft formal context $T = (G, M, R, F)$.

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