A Multi-Level Multi-Objective Quadratic Programming Problem with Fuzzy Parameters on Objective Functions

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ABSTRACT

This paper proposes analgorithm to solve multi-level multi-objective quadratic programming problemwith fuzzy parameters in the objective functions, This algorithm uses the tolerance membership function conceptsand multi-objective optimization at each level to develop a fuzzy Max-Min decision model for generating satisfactory solution after applying linear rankingmethod ontrapezoidal fuzzy numbers in the objective functions, An illustrative example is included to explain the results.

Keywords

Multi-Level Programming,Trapezoidal Fuzzy Numbers, linear Ranking Methods, Fuzzy approach.

Academic Discipline And Sub-Disciplines

Computers & Technology
Management & Information Technology

TYPE (METHOD/APPROACH)

Multi- Objective Non-linear Programming, Fuzzy programming techniques, Quadratic programming.

1. INTRODUCTION

Multi-level programming (MLP) techniques are developed to solve decentralized problems that contain multiple decision-makers in hierarchical organization, where each unit or department independently seeks its own interest, but is affected by the actions of other unit through externalities.Three level programming is a class of Multi-level programming problem in which there are three independent decision-makers.The field of multi-level programming which defines the art and science of making such decisions studied in [2, 3, and 4].

Fuzzy approach uses the concept of tolerance membership to develop a fuzzy max-min decisionModel for generating Pareto optimal (satisfactory) solution [1 and 3]

In [3] Osman et al. presented the concepts of tolerance membership function and multi-objective optimization at each level to develop a fuzzy max-min decision model for generating (satisfactory) solution for three level multi-objective programming problem.

Recently several linear or nonlinear programming problems with fuzzy parameters on objective functions and their solution method have been presented, such as, in [4, 5, 6,7, 8, 9, 10and11]. In [4] Under the rules of simplex technique and the operations on trapezoidal fuzzy numbersEmamet al. ,suggested a new solution method to solve bi-level linear fractional integer programming problem with trapezoidal fuzzy numbers in the objective functions of the two levels .In [5]Dashetal. aimed to present a method in which a fuzzy multi objective nonlinear programming problem is reduced to crisp using ranking function and then the crisp problem is solved by fuzzy programming technique. In [6]Nasseri defined a quadratic programming problem with trapezoidal and/or triangularfuzzy numbers in the cost coefficients,constraint coefficients, and right-hand sides then used linear ranking method to solve the problem.

2. PROBLEM FORMULATION AND SOLUTION CONCEPT

Let $x_i \in R^n$, (i = 1, 2, 3),be a vector of variables which indicates the first decision level's choice, thesecond decision level's choice and the third decision level's choice and$F_i : R^n \rightarrow R^{N_t}$(i = 1, 2, 3),be the first level objective function, the second level objective function and the third level objective function, respectively. Assume thatthe first level decision maker is (FLDM),the secondlevel decision maker is(SLDM) and the third level decision maker is(TLDM).$N_1, N_2 and N_3 \geq 2$,the FLDM, SLDM, and TLDM haveN_1, N_2 and N_3objective functions, respectively.Let$G$be the set of feasible choices $\{(x_1, x_2, x_3)\}$. Therefore a(MLMOQPP) with fuzzy parameters in the objective functionsmay be formulated as follows:

[**1st level**]

$$\max_{x_1} F_1(\overline{x}, \tilde{q}, \tilde{C}) = \max_{x_1}\left[\left(\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\tilde{q}_{ij}x_ix_j + \sum_{j=1}^{n}\tilde{C}_jx_j\right),..,f_{1N_1}\left(\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\tilde{q}_{ij}x_ix_j + \sum_{j=1}^{n}\tilde{C}_jx_j\right)\right] \tag{1}$$

Where x_2, x_3 solve

[**2nd level**]

$$\max_{x_2} F_2(\overline{x}, \tilde{q}, \tilde{C}) = \max_{x_2}\left[\left(\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\tilde{q}_{ij}x_ix_j + \sum_{j=1}^{n}\tilde{C}_jx_j\right),...,f_{2N_2}\left(\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\tilde{q}_{ij}x_ix_j + \sum_{j=1}^{n}\tilde{C}_jx_j\right)\right] \tag{2}$$

Where x_3 solves

[**3rd level**]

$$\max_{x_3} F_3(\overline{x}, \tilde{q}, \tilde{C}) = \max_{x_3}\left[\left(\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\tilde{q}_{ij}x_ix_j + \sum_{j=1}^{n}\tilde{C}_jx_j\right),...,f_{3N_3}\left(\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\tilde{q}_{ij}x_ix_j + \sum_{j=1}^{n}\tilde{C}_jx_j\right)\right] \tag{3}$$

Subject to:

$$G: \{(\overline{x})|g_i(\overline{x}) \le 0, i = 1,2,\dots,m\}, \overline{x} = (x_1, x_2, x_3) \in R^{n_1+n_2+n_3} \tag{4}$$

Where $F_i(\overline{x}, \tilde{q}, \tilde{C})$, is a multi-level multi-objective quadratic programming problem with fuzzy parameters in the objective functions.

Definition 1 [3]:

Let G_1, G_2, G_3 be the feasible regions of FLDM, SLDM and TLDM, respectively. For any $(x_1 \in G_1 = \{x_1/ (x_1, x_2, x_3)\}. \in G_1\})$ given by FLDM, and $(x_2 \in G_2 = \{x_2/ (x_1, x_2, x_3 \in G_2\})$ given by SLDM, if the decision-making variable $(x_3 \in G_3 = \{x_3/ (x_1, x_2, x_3) \in G_3\})$ is the optimal solution of the TLDM, then (x_1, x_2, x_3) is a feasible solution of the(MLMOQPP) with fuzzy parameters in the objective functions.

Definition 2[3]:

If (x_1^*, x_2^*, x_3^*) is a feasible solution of the (MLMOQPP) with fuzzy parameters in the objective functions(1)-(4); no other feasible solution $(x_1, x_2, x_3) \in G$ exists, such that $f_{1i}(x_1^*, x_2^*, x_3^*) \le f_{1i}(x_1, x_2, x_3)$, with at least one $(i=1,2,..,K_i)$; so (x_1^*, x_2^*, x_3^*) is the optimal solutions of the (MLMOQPP) with fuzzy parameters in the objective functions.

3. RANKING METHOD

To solve (MLMOQPP) with fuzzy parameters in the objective functions a linear ranking method technique is used to convert fuzzy number form into equivalent crisp form.

Definition 3.1[10]:

If $(\breve{A})$=(a, b, c, d)$\in$ F(R), then a linear ranking function is defined as $\Re(\breve{A})= a + b + \frac{1}{2}(d - c)$ $\tag{5}$

Definition 3.2[10]:

$\tilde{A}(a_1, b_1, c_1, d_1), \breve{B}(a_2, b_2, c_2, d_2)$ are two trapezoidal fuzzy numbers and x$\in$R. Ranking function is a convenient method for comparing the fuzzy numbers which is a map from F(R) into the real line. So, the orders on F(R) as follow:

1. $\tilde{A} \ge \tilde{B}$ if and only if $\Re(\tilde{A}) \ge \Re(\tilde{B})$.
2. $\tilde{A} > \tilde{B}$ if and only if $\Re(\tilde{A}) > R(\tilde{B})$.
3. $\tilde{A} = \tilde{B}$ if and only if $\Re(\tilde{A}) = \Re(\tilde{B})$.

Where $\tilde{A} and \tilde{B}$ are in F(R).

Now after applying linear ranking method the problem will be formulated as follow:

[$1^{st} level$]

$$\max_{x_1} F_1(x_1, x_2, x_3) = \max_{x_1} \left[\left(\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} q_{ij}x_ix_j + \sum_{j=1}^{n} C_jx_j \right), \dots, f_{1N_1}\left(\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} q_{ij}x_ix_j + \sum_{j=1}^{n} C_jx_j \right) \right] \qquad (6)$$

Where x_2, x_3 solve

[$2^{nd} level$]

$$\max_{x_2} F_2(x_1, x_2, x_3) = \max_{x_2} \left[\left(\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} q_{ij}x_ix_j + \sum_{j=1}^{n} C_jx_j \right), \dots, f_{2N_2}\left(\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} q_{ij}x_ix_j + \sum_{j=1}^{n} C_jx_j \right) \right] \qquad (7)$$

Where x_3 solves

[$3^{rd} level$]

$$\max_{x_3} F_2(x_1, x_2, x_3) = \max_{x_3} \left[\left(\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} q_{ij}x_ix_j + \sum_{j=1}^{n} C_jx_j \right), \dots, f_{3N_3}\left(\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} q_{ij}x_ix_j + \sum_{j=1}^{n} C_jx_j \right) \right] \qquad (8)$$

Subject to:

$$G: \{(\bar{x})|g_i(\bar{x}) \le 0, i = 1,2,\dots,m\}, \bar{x} = (x_1, x_2, x_3) \in R^{n_1+n_2+n_3} \qquad (4)$$

4. FUZZY APPROACH FOR SOLVING (MLMOQPP)

To solve the (MLMOQPP) by using fuzzy approach, first the satisfactory solution that is acceptable for the FLDM is obtained, then the FLDM's decision variables and goals with some leeway are given to the SLDM to seek the satisfactory solution according to him/her, then the SLDM's decision variables and goals with some leeway are given to the TLDM to seek the satisfactory solution according to him/her and finally to arrive at the solution which is closest to the optimal solution of the FLDM.[3]

4.1. The problem ofthe FLDM.

The problem of theFLDM is solved individually;FLDM'soptimal solution is achieved by applying the following steps, firstby obtaining the best and the worst solutions oftheProblem

$$(f_{11}^{*}, \dots\ f_{1k}^{*}) (f_{11}^{-}, \dots\ f_{1k}^{-}), \text{ where } f_{1k}^{*} = \underset{x \in G}{Max} f_{1k}(\bar{x}), f_{1k}^{-}(\bar{x}) = \underset{x \in G}{Min} f_{1k}(\bar{x}), k = 1,2,\dots N_1. \qquad (9)$$

Second Usingthe value of (f_{1k}^{*}, f_{1k}^{-}) to build membership functions as follows:

$$\mu_{f_{1k}}\left[f_{1k}(\bar{x})\right] = \begin{cases} 1 & if & f_{1k}(\bar{x}) > f_{1k}^{*} \\ \dfrac{f_{1k}(\bar{x}) - f_{1k}^{-}}{f_{1k}^{*} - f_{1k}^{-}} & if & f_{1k}^{-} \le f_{1k}(\bar{x}) \le f_{1K}^{*} \\ 0 & if & f_{1k}^{-} \ge f_{1k}(\bar{x}) \\ & & k = 1,2,\dots,N_1 \end{cases} \qquad (10)$$

Third by solving the Tchebycheff problem as follow:

$$Max\,\lambda,$$

(11)

Subject to:

$$\{\bar{x} \in G,$$

$$\mu_{f_{1k}}\left[f_{1k}(\bar{x})\right] \geq \lambda, k = 1,2,\ldots,N_1,$$

$$\lambda \in [0,1]\}$$

Whose solution is assumed to be: [$x_1^F, x_2^F, x_3^{F\cdot}, f_{1k}^F, k = 1,2,\ldots,N, \lambda^F$]   (satisfactory level).

Now the SLDM and TLDM do the same action like the FLDM till they obtains their optimal solutions as
[$x_1^S, x_2^S, x_3^S, f_{2q}^S, q = 1,2,\ldots,N, \beta^s$] and [$x_1^t, x_2^t, x_3^t, f_{2r}^t, r = 1,2,\ldots,N, \gamma^t$]

Now the solution of the three level decision makers is disclosed. However, three solutions are usually different because of nature between three levels objective functions.

The FLDM knows that using the optimal decisions x_1^F as a control factors for the SLDM are not practical. It is more reasonable to have some tolerance that gives the SLDM an extent feasible region to search for his/her optimal solution, and reduce searching time or interactions, also the SLDM do the same action with the TLDM. In this way, the range of decision variables x_1, x_2 should be around x_1^F , x_2^S with maximum tolerance t_1, t_2 and the following membership function specify x_1^F , x_2^S as:

$$\mu_{x_1}(x_1) = \begin{cases} \dfrac{x_1 - (x_1^F - t_1)}{t_1} & x_1^F - t_1 \leq x_1 \leq x_1^F \\[2ex] \dfrac{(x_1^F + t_1) - x_1}{t_1} & x_1^F \leq x_1 \leq x_1^F + t_1 \end{cases}$$

(12)

$$\mu_{x_2}(x_2) = \begin{cases} \dfrac{x_2 - (x_2^S - t_2)}{t_1} & x_2^S - t_2 \leq x_2 \leq x_2^S \\[2ex] \dfrac{(x_2^S + t_2) - x_2}{t_2} & x_2^S \leq x_2 \leq x_2^S + t_2 \end{cases}$$

(13)

The FLDM goal may be reasonably consider all $f_{1k} \geq f_{1k}^F\ k = 1,2,\ldots N_1$ are absolutely acceptable and f_{1k}

$< f_{1k}' = f_{1k}\,(x_1^S, x_2^S, x_3^S)$, $k = 1,2,\ldots N_1$ are absolutely unacceptable, and that the preference with[$f_{1k}^F, f_{1k}', k = 1,2,\ldots N_1$] is linearly increasing. This is due to the fact that the SLDM obtained the optimum at (x_1^S, x_2^S, x_3^S) which in turn provides the FLDM the objective function values f_{1k}' , makes any $f_{1k}^F, f_{1k}', k = 1,2,\ldots N_1$, unattractive in practice, The following membership functions of the FLDM can be stated as

$$\mu_{f_{1k}}'\left[f_{1k}(\bar{x})\right] = \begin{cases} 1 & if\ f_{1k} > f_{1k}^F(x) \\[1ex] \dfrac{f_{1k}(\bar{x}) - f_{1k}'}{f_{1k}^F - f_{1k}'} & if\ f_{1k}' \leq f_{1k}(x) \leq f_{1K}^F \\[1ex] 0 & if\ f_{1k}(x) \geq f_{1k}' \end{cases}$$

(14)

Second, the SLDM goal may reasonably consider all $f_{2q} \geq f_{2q}^S \geq q = 1,2,\ldots N_2$ are absolutely acceptable and $f_{2q} < f_{2q}' = f_{2q}\,(x_1^T, x_2^T, x_3^T)$, $q = 1,2,\ldots N_1$ are absolutely unacceptable, and that the preference with[$f_{2q}^S, f_{2q}', q = 1,2,\ldots N_2$] is linearly

increasing. This is due to the fact that the TLDM obtained the optimum at (x_1^T, x_2^T, x_3^T) which in turn provides the SLDM the objective function values f'_{2q}, makes any [$f_{2q}^S, f'_{2q}, q = 1,2,\ldots N_2$], unattractive in practice, The following membership functions of the SLDM can be stated as

$$\mu'_{f_{2q}}\left[f_{2q}(\bar{x})\right] = \begin{cases} 1 & if\ f_{2q} > f_{2q}^S(x) \\ \dfrac{f_{2q}(\bar{x}) - f'_{2q}}{f_{2q}^S - f'_{2q}} & if\ f'_{2q} \le f_{2q}(x) \le f_{2q}^S \\ 0 & if\ f_{2q}(x) \ge f'_{2q} \end{cases} \tag{15}$$

Third, the TLDM may be willing to build a membership function for his/her objective functions, so that he/she can rate the satisfaction of each potential solution. In this way, the TLDM has the following membership functions for his/her goals:

$$\mu'_{f_{3r}}\left[f_{3r}(\bar{x})\right] = \begin{cases} 1 & if\ f_{3r} > f_{3r}^T(x) \\ \dfrac{f_{3r}(\bar{x}) - f'_{3r}}{f_{3r}^T - f'_{3r}} & if\ f'_{3r} \le f_{3r}(x) \le f_{3r}^T \\ 0 & if\ f_{3r}(x) \ge f'_{3r} \end{cases} \tag{16}$$

Finally, in order to generate the satisfactory solution, which is also a Pareto optimal solution with overall satisfaction for all DMs, theTchebycheff problem can be solved as follow:

$$Max\,\delta, \tag{17}$$

Subject to:
$$\{\bar{x} \in G,$$
$$\mu(x_i) \ge \delta,$$
$$\mu_{f_{1k}}\left[f_{1k}(\bar{x})\right] \ge \delta, (k = 1,2,\ldots,N_1),$$
$$\mu_{f_{2q}}\left[f_{2q}(\bar{x})\right] \ge \delta, (q = 1,2,\ldots,N_2),$$
$$\mu_{f_{3r}}\left[f_{3r}(\bar{x})\right] \ge \delta, (r = 1,2,\ldots,N_3),$$
$$i = 1,2.$$
$$\delta \in [0,1]\}.$$

5. AN ALGORITHM

In this section an algorithm is presented to solve (MLMOQPP) with fuzzy parameters in the objective functions, the algorithm is illustrated in the following series steps:

Step 1: Compute $\Re(\tilde{A})$for all the coefficients ofthe (MLMOQPP) with fuzzy number in the objective functions (1) - (3), where $\tilde{A}$ is a trapezoidal fuzzy number.

Step 2: Convert the (MLMOQPP) with fuzzy number in the objective functions (1) - (3)from the fuzzy form to the crisp form.

Step 3: Formulate the (MLMOQPP) (6) - (8).

Step 4: Use fuzzy approachto solve the (MLMOQPP).

Step 5: The FLDM finds the individual best solutions f_{1k}^* and individual worst f_{1k}^- for each objective of the FLDM.

Step 6: State membership functions of the FLDM, $\mu_{f_{1k}}\left[f_{1k}(\bar{x})\right]$

Step 7: Solve the Tchebycheff problem of the FLDM.

Step 8: Obtain satisfactory level of FLDM $x_1^F, x_2^F, x_3^F, f_{1k}^F, k = 1,2,\ldots N, \lambda^F$.

Step 9: The SLDM finds the individual best solutions f_{2q}^* and individual worst f_{2q}^- for each objective of the SLDM.

I S S N   2 2 7 7 - 3 0 6 1
V o l u m e   1 5   N u m b e r   5
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Step 10: State membership functions of the SLDM, $\mu_{f_{2q}}\left[f_{2q}(\bar{x})\right]$.

Step 11: Solve the Tchebycheff problem of the SLDM.

Step 12: Obtain satisfactory level of SLDM[$x_1^S, x_2^S, x_3^S, f_{2q}^S, q = 1,2,...,N,\ \beta^s$].

Step 13: The TLDM finds the individual best solutions f_{3r}^* and individual worst f_{3r}^- for each objective of the TLDM.

Step 14: State membership functions of the TLDM $\mu_{f_{3r}}\left[f_{3r}(\bar{x})\right]$.

Step 15: Solve the Tchebycheff problem of the TLDM.

Step 16: Obtain satisfactory level of TLDM [$x_1^t, x_2^t, x_3^t, f_{2r}^t, r = 1,2,...,N,\ \gamma^t$]

.**Step 17:** Set t_1, t_2 then calculate $\mu_{x_1}(x_1)$, $\mu_{x_2}(x_2)$

Step 18: state membership function $\mu'_{f_{1k}}\left[f_{1k}(\bar{x})\right]$, $\mu'_{f_{2q}}\left[f_{2q}(\bar{x})\right]$, $\mu'_{f_{3r}}\left[f_{3r}(\bar{x})\right]$

Step 19: Solve the Tchebycheff problem for all decision makers problem.

Step 20: If $\delta < 0.5$, decrease tolerance value t_1, t_2, then go to step 17, otherwise go to step 21.

Step 21: A compromise solution X^0 of the (MLMOQP) problem is obtained and δ is overall satisfaction for all decision - makers.

5.1.A flowchart:

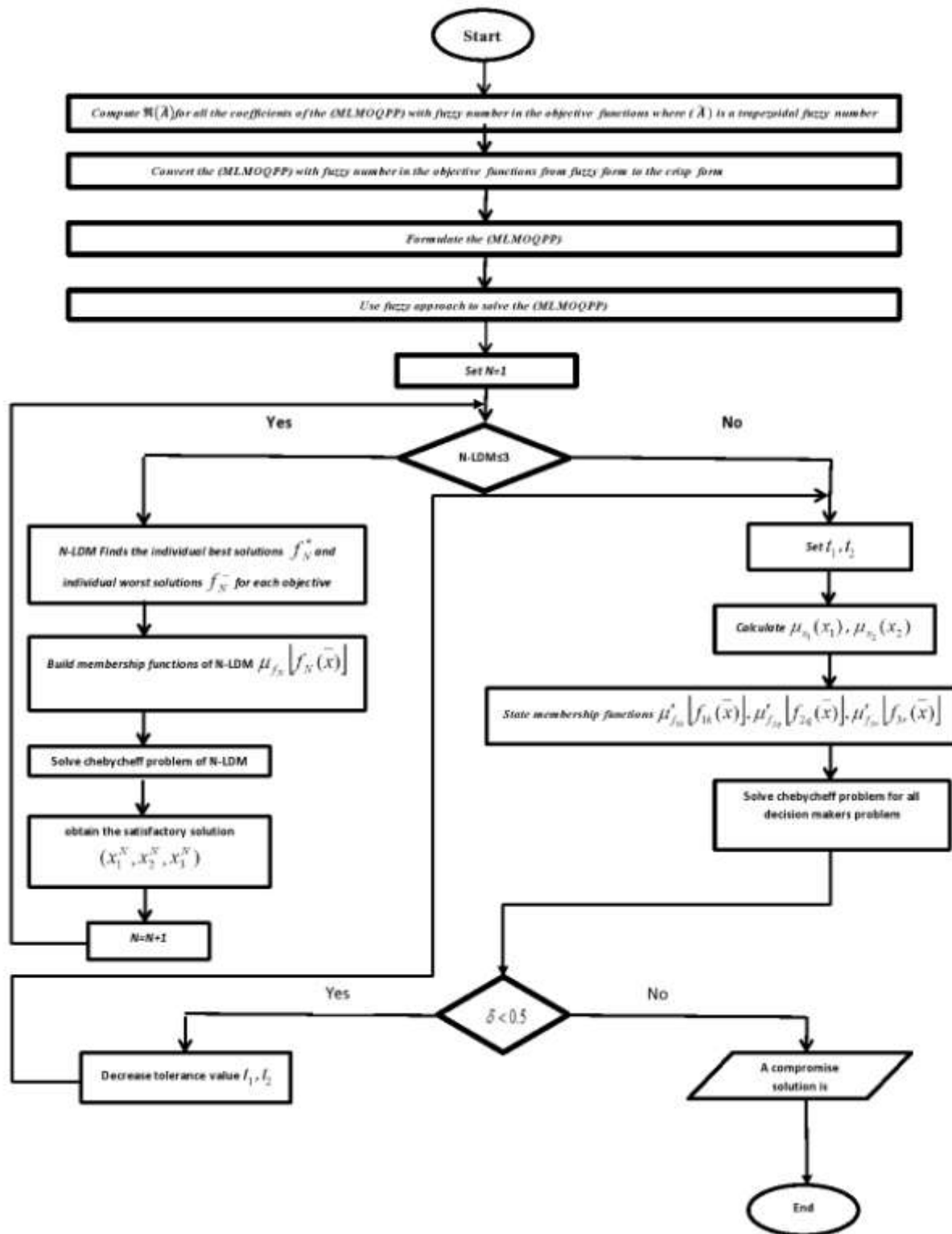A flowchart to explain the suggested algorithm is described as follow:

I S S N 2 2 7 7 - 3 0 6 1
V o l u m e 1 5 N u m b e r 5
I n t e r n a t i o n a l J o u r n a l o f C o m p u t e r s a n d T e c h n o l o g y

Fig1: An algorithm for solving the (MLMOQPP) with fuzzy number in the objective functions.

6. NUMERICAL EXAMPLE

[$1^{st} level$]

$$max\ F_1(x_1,x_2,x_3) \quad = max \quad [3(1,3,2,4)x_1^2 + x_2^2 + x_3 \quad , \quad 4(1,2,4,1)x_1^2 + 3x_2^2]$$

$$x_1 \qquad\qquad x_1$$

Where x_2, x_3 solves

[$2^{nd} level$]

$$max\ F_2(x_1,x_2,x_3) \quad = max_{x_2}[x_1^2 + 3x_2^2 \qquad , \qquad 2x_1^2 + (2,4,6,8)x_2^2 + 3x_3]$$

$$x_2$$

Where x_3 solves

[$3^{rd} level$]

$$max\ F_3(x_1,x_2,x_3)_{x_3} = max_{x_3}[x_1 + x_2 + 3(1,3,2,4)x_3^2 \quad , \quad 2x_1^2 + x_2^2 + 2(3, 4, 5, 6)x_3^2]$$

Subject to

$(x_1,\ x_2,\ x_3) \in G =$

$\{(x_1,\ x_2,\ x_3)$
$x_1 + x_2 + x_3 \le 5$
$x_1 + 2x_2 + x_3 \le 10$
$2x_1 + x_2 + x_3 \le 14$

$x_1,x_2,x_3 \ge 0\}$

Firstly, by using equation (5)all the coefficients ofthe problem are computed so the problem is convertedfrom the fuzzy form to the crisp formthen the problem is formulated as follow:

[$1^{st} level$]

$$max\ F_1(x_1,x_2,x_3) \quad = max \quad [15x_1^2 + x_2^2 + x_3 \quad , \quad 6x_1^2 + 3x_2^2]$$

$$x_1 \qquad\qquad x_1$$

Where x_2 solves

[$2^{nd} level$]

$$max\ F_2(x_1,x_2,x_3) \quad = max_{x_2}[x_1^2 + 3x_2^2 \quad , \quad 2x_1^2 + 7x_2^2 + 3x_3]$$

$$x_2$$

Where x_3 solves

[$3^{rd} level$]

$$max\ F_3(x_1,x_2,x_3)_{x_3} = max_{x_3}[x_1 + x_2 + 15x_3^2 \quad 2x_1^2 + x_2^2 + 15x_3^2\ ]$$

Subject to

$(x_1,\ x_2,\ x_3) \in G =$

$\{(x_1,\ x_2,\ x_3)$
$x_1 + x_2 + x_3 \le 5$
$x_1 + 2x_2 + x_3 \le 10$
$2x_1 + x_2 + x_3 \le 14$

$x_1,x_2,x_3 \ge 0\}$

Secondly, by using fuzzy approach, TheFLDM solves his/her problem as follows:

[$1^{st} level$]

$$max\ F_1(x_1,x_2,x_3) \quad = max \quad [15x_1^2 + x_2^2 + x_3 \quad , \quad 6x_1^2 + 3x_2^2]$$

$$x_1 \qquad\qquad x_1$$

Subject to

$(x_1,\ x_2,\ x_3) \in G =$

$\{(x_1,\ x_2,\ x_3)$
$x_1 + x_2 + x_3 \le 5$

$$x_1 \ + 2x_2 \ + x_3 \le 10$$
$$2x_1 \ + x_2 \ + x_3 \le 14$$

$x_1 \ , x_2 \ , x_3 \ge 0\}$

By solving equation (9), $(f_{11}^*, f_{12}^*) = (375,150)$, $(f_{11}^-, f_{12}^-) = (-.18,-.19)$, thenequation (10) and then equation (11)

$Max\lambda,$

Subject to:{

$(x_1, \ x_2, \ x_3) \in G,$

$\lambda \in [0,.2],$

$15x_1^2 + x_2^2 + x_3 \ -375.18 \ \lambda >= -.18,$

$6x_1^2 + 3x_2^2 -150.19 \ \lambda >= - 0.19\}$

Whose solution is assumed to be:

$\left(x_1^F, x_2^F, x_3^F\right) = (2.60, 0.67, 1.23)$, $(f_{11}^F, f_{12}^F) = (103.07, 91.90)$, $\lambda = 0.2$

The SLDM solves his/her problem exactly like FLDM as follow:

[$2^{nd} level$]

$$max\, F_2(x_1, x_2, x_3) \ = max_{x_2}[x_1^2 + 3\,x_2^2 \ , \quad 2x_1^2 + 7x_2^2 + 3x_3]$$
$$x_2$$

Subject to:

$(x_1, \ x_2, \ x_3) \in G = \{(x_1, \ x_2, \ x_3)$

$$x_1 \ + x_2 \ + x_3 \le 5$$
$$x_1 \ + 2x_2 \ + x_3 \le 10$$
$$2x_1 \ + x_2 \ + x_3 \le 14$$

$x_1 \ , x_2 \ , x_3 \ge 0\}$

$(f_{21}^*, f_{22}^*) = (75,175).(f_{21}^-, f_{22}^-) = (-0.16,-0.19)$

$Max\beta,$

Subject to :{

$(x_1, \ x_2, \ x_3) \in G,$

$\beta \in [0,0.1].$

$x_1^2 + 3\,x_2^2 - \ 75.16\beta \ge -0.16$

$2x_1^2 + 13x_2^2 + 3x_3 - 175.19 \ \beta \ \ge -0.19\}$

Whose solution is assumed to be:

$\left(x_1^S, x_2^S, x_3^S\right) = (1.74, 1.23, 0.39)$, $(f_{21}^S, f_{22}^S) = (7.56, 17.81)$, $\beta = 0.1$

The TLDM solves his/her problem exactly like SLDMas follow:

[$3^{rd} level$]

$max\, F_3(x_1, x_2, x_3)_{x_3} = max_{x_3}[x_1 + x_2 + 15x_3^2 \ , \quad 2\,x_1^2 + x_2^2 + 15x_3^2]$

Subject to:

$(x_1, \ x_2, \ x_3) \in G = \qquad \{(x_1\, x_2, x_3)$

$$x_1 \ + x_2 \ + x_3 \le 5$$
$$x_1 \ + 2x_2 \ + x_3 \le 10$$
$$2x_1 \ + x \ + x_3 \le 14$$

$x_1 \ , x_2 \ , x_3 \ge 0\}$

$(f_{31}^*, f_{32}^*) = (375,375)$, $(f_{31}^-, f_{32}^-) = (0,0)$

$Max\gamma,$

Subject to :{

$(x_1, \ x_2, \ x_3) \in G,$

I S S N   2 2 7 7 - 3 0 6 1
V o l u m e   1 5   N u m b e r   5
I n t e r n a t i o n a l   J o u r n a l   o f   C o m p u t e r s   a n d   T e c h n o l o g y

$x_1 + x_2 + 15x_3^2\text{-}375\gamma \geq 0$

$2*x_1^2 + x_2^2 + 15x_3^2\text{-}375\gamma \geq 0$

$\gamma \in [0,0.1].\ \}$

Whose solution is assumed to be :

$\left(x_1^T, x_2^T, x_3^T\right) = (0.70, 1.23, 1.57), (f_{11}^T, f_{12}^T) = (38.90, 39.46),\ \gamma = 0.1$

<u>Finally</u>

1- Assume the FLDM's control decision is around 0 with tolerance 1.

2- Assume the SLDM's control decision is around 0 with the tolerance 1.

3- By using (12)-(17)theTchebycheff problem can be solved as follow:

$Max\delta,$

Subject to:{

$\left(x_1,\ x_2,\ x_3\right) \in G,$

$x_1 - \delta \geq 1.6$

$-x_1 - \delta \geq -3.6$

$x_2 - \delta \geq 0.23$

$-x_2 - \delta \geq -2.23$

$15x_1^2 + x_2^2 + x_3 + 55.76\delta \geq 47.31$

$6x_1^2 + 3x_2^2 + 19.2\delta \geq 22.70$

$x_1^2 + 3x_2^2 + .54\delta \geq 8.1$

$2x_1^2 + 7x_2^2 + 3x_3 + 2.54\delta \geq 20.35$

$x_1 + x_2 + 15x_3^2\text{-}33.64\delta \geq 5.25$

$2x_1^2 + x_2^2 + 15x_3^2\text{-}29.62\delta \geq 9.84$

$\delta \in [.5,1]\}$

Whose is Compromise solution X^0 = (2.46, 1.09, 1.43), δ =0.87.

$(f_{11}^0, f_{12}^0) = (93.39, 39.87)$

$(f_{21}^0, f_{22}^0) = (9.61, 24.70)$

$(f_{31}^0, f_{32}^0) = (34.22, 43.96)$

7. CONCLUSION

This paper proposed an algorithm to solve Multi-level multi-objective Quadratic programming problem, with fuzzy parameters in the objective functions, This algorithm used the concepts of tolerance membership function and multi-objective optimization at each level to develop a fuzzy Max-Min decision model for generating satisfactory solution after applying linear Ranking Method on trapezoidal fuzzy numbers in the objective function , This algorithm can be applied to problems when the fuzzy numbers in the constraint or in both the objective function and the constraint

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