



# Impact of using Infinity-Norm with Initial Radius on Performance and Complexity of SD Algorithm in MIMO systems

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## ABSTRACT

In recent years, the iterative decoding techniques have played a role in improving the performance (e.g., bit error rate) and reducing the complexity of various digital communication systems. Techniques of Multiple-Input Multiple-Output (MIMO) are the main technology to enhance and achieve high-speed, high data rates, improved reliability and coverage in wireless communications. The modern wireless communications require a low complexity system for detection, since a high CPU processing involves more energy consumption and thus less flexibility in mobility terms. The sphere decoding (SD) technique proposed to solve this problem, such as an efficient algorithm. The norm-2 or  $\ell^2$ -norm (Euclidean metric) considered as a traditional norm that is used to achieve the tree traversal stage in SD algorithm. This work is divided into two parts; Firstly, we propose to using Infinity-Norm or  $\ell^\infty$ -norm instead  $\ell^2$ -norm to decreases the hardware complexity of SD with a loss of performance is negligible, the simulation results show that the proposed  $\ell^\infty$ -norm SD needs 14.5% to 5.9% fewer complexities than  $\ell^2$ -norm SD. Secondly, we are investigating the impact of choosing initial radius on the performance and complexity of SD algorithm, we can conclude from the simulation results, that gain a better performance require increasing in the initial radius of an SD algorithm from d1 ( $\gamma = 2$ ) to d3 ( $\gamma = 8$ ), and this mean addition more complexity due to the tradeoff between performance and complexity.

## Indexing terms/Keywords

MIMO detection, SD algorithm, Infinity-norm, Euclidean metric, Initial Radius of sphere.

## Academic Discipline And Sub-Disciplines

Electronic and Information Eng. Communication systems

## SUBJECT CLASSIFICATION

Communication systems; MIMO detection techniques

## TYPE (METHOD/APPROACH)

Quasi-Experimental; Literary Analysis

## 1. INTRODUCTION

In the future, communication systems need to use the available spectrum with large efficient in order to increase throughput. In wireless communications, spectral efficiency can be increased by using multiple transmit / receive antennas like multiple input multiple outputs (MIMO)[1] systems. However, whereas the capacity of those MIMO channels increases linearly with the number of antennas, the complexity of detection schemes increases exponentially. MIMO is one of the important technologies to gathering the demands for higher spectral efficiency and improved quality-of-service (QoS) in next generation wireless communication systems[2]. MIMO communication systems have three main advantages diversity gain, array gain, and multiplexing gain that make it a powerful, provides reliable, high-speed, and bandwidth-efficient data transmission through iterative processing with channel decoding in detection process[3]. In MIMO systems, the main challenge is how to separate the spatially multiplexed data streams while ensuring high performance and low computational complexity. Various detection algorithms for MIMO systems, exhibit different tradeoffs between performance and complexity, have been proposed in many kinds of literature [4-13]. The computational complexity of an MIMO detection algorithms depends on the number of spatially multiplexed data streams (number of transmit antennas) and the symbol constellation size, but frequently on the instantaneous MIMO channel realization and the signal-to-noise ratio (SNR)[14]. A novel and efficient MIMO detection algorithm for any wireless communication systems must include some important features such as low-complexity, near-optimal performance and robust scheme. The maximum likelihood detector (MLD) [15] can present outstanding performance; but, it suffers from high computational complexity in practical implementation especially when increasing the number of transmit antennas to achieve a good transmission capacity in MIMO systems. Sphere detection /decoder (SD) [4, 11, 16-18]was investigated to achieve the maximum likelihood (ML) performance by using reliable radius. The idea of SD was introduced in [19] and it has been furthermore debated in various researches[18, 20]. Key issues affecting the SD are: *First*, How to select the search radius. If the radius is too large, that mean it contains too much candidate points inside the sphere. So the computational complexity will reach to the exponential complexity of the MLD. If the radius is too small, the sphere may be within a grid are not included, then the SD algorithm will not be a reasonable solution. *Second*, How to determine whether a candidate point in the sphere. This needs to calculate the distance between each candidate point and the receive vector, then this approach is not ideal, because of needing to search for all the candidate points, and so the resulting calculation is exponential[18, 21]. An



important feature is that the SD algorithms perform the detection layer by layer, and the layers are equal to the number of receive antennas in the MIMO system[18]. The enumeration process is considered the important part in the SD algorithms to reduce complexity. The process of selecting the candidate points can increase or decrease the number of operations performed, also the number of chosen candidate points, and the propagated error from layer to another. There are two main forms of enumeration processes: Schnorr - Euchner [22] and Fincke – Pohst [23]. Thus, the alternative to reduce complexity is mean reducing the complexity of the enumeration process, by setting limitations on the number of candidate points that are selected as candidates. The concept of complexity, defined as the number of floating point operations (additions, multiplications etc.) which are required to compute the estimated transmit vectors or the running time of the detection algorithm when implemented on some specific programs[24]. In this work and in simulation results, we depend on running time of the detection algorithm to calculate the complexity. In SD algorithm while conducting tree traversal using the  $\ell^2$ -norm is optimum, it was observed in[25] that performing tree traversal based on the  $\ell^\infty$ -norm instead of using the  $\ell^2$ -norm leads to significantly reduced VLSI implementation complexity at only a marginal performance loss. The area timing product from SD with  $\ell^\infty$ -norm that arrive at a factor lower five times than those of SD with  $\ell^2$ -norm. These good savings comes from a reduction in the length of the critical path and the silicon area of the circuit [26]. Therefore, SD with  $\ell^\infty$ -norm showed a promising approach to near-optimal MIMO detection at low hardware complexity. Therefore, for these reasons that we mentioned, we decide to study the  $\ell^\infty$ -norm instead of  $\ell^2$ -norm in SD algorithm. In SD algorithm, the choice of the initial search radius is one of the important drawbacks because this parameter is important for identification of candidate nodes in the sphere, where its plays a big role in increasing or decreasing the complexity of SD algorithm. There are several methods to deciding the initial radius, but so far not exist a general method that works well in all different applications or channel conditions [27].

The rest of this paper is organized as follows; The MIMO wireless communication system model and  $\ell^2$ -norm SD are described in Section 2. The  $\ell^\infty$ -norm SD calculation appears in Subsection 2.1. The impact of the initial radius on SD algorithm explained in the Subsection 2.2. Section 3. Shows the simulation results. Finally, section 4. Shows the conclusions of the paper.

## 2. System Model and SD algorithm:

We consider a  $(N_t \times N_r)$  uncoded complex-valued MIMO communication system with M-QAM modulation,  $N_t$  is the transmit antennas, and  $N_r$  is the receive antennas, moreover assume  $(N_r \geq N_t)$  throughout the paper the equivalent discrete-time baseband system model can be represented as

$$\bar{y} = \bar{H}\bar{x} + \bar{n} \dots\dots (1),$$

where  $\bar{y}$  is  $(N_r \times 1)$  received symbol vector,  $\bar{x}$  is  $(N_t \times 1)$  transmitted symbol vector, the elements of  $\bar{x}$  is independent of each other and is drawn from a complex constellation  $\Omega$  with M bits per symbol i.e.,  $|\Omega| = 2^M$ , and the set of all possible transmitted vector symbols is denoted by  $\Omega^{N_t}$ ,  $\bar{H}$  is the  $(N_r \times N_t)$  Rayleigh fading channel matrix, and  $\bar{n}$  is the  $(N_r \times 1)$  independent and identically distributed (i.i.d.) complex zero-mean additive white Gaussian noise (AWGN) vector and variance  $\sigma_n^2$ , the SNR of the system is given by  $SNR = 1/\sigma_n^2$ . We assume that the receiver has gained knowledge of the channel matrix.

To take advantage of the QR Decomposition, the complex-valued system model in (1) can be transformed into the real-valued system model as:

$$y = Hx + n \dots\dots (2)$$

$$\begin{bmatrix} \Re[y] \\ \Im[y] \end{bmatrix} = \begin{bmatrix} \Re[H] & -\Im[H] \\ \Im[H] & \Re[H] \end{bmatrix} \begin{bmatrix} \Re[x] \\ \Im[x] \end{bmatrix} + \begin{bmatrix} \Re[n] \\ \Im[n] \end{bmatrix} \dots\dots (3)$$

where  $\Re[\cdot]$  and  $\Im[\cdot]$  denote the real and imaginary parts of  $[\cdot]$  respectively, and the dimension of real-valued system model becomes equal to  $N=2N_r=2N_t$ . In MIMO system, the symbol detection process depends on the value of  $y$  and  $H$  to find the estimated  $x$ , the optimal estimate that is constrained with initial search radius ( $d$ ) is the ML solution as:

$$\underset{\hat{x} \in A^N}{\operatorname{argmin}} \| y - H\hat{x} \|^2 \leq d^2 \dots\dots (4)$$

where A is the constellation in the real-valued system model.

By applying the QR-decomposition to  $H$ , (4) can be simplified more as:

$$\underset{\hat{x} \in A^N}{\operatorname{argmin}} \| y - QR\hat{x} \|^2 = \underset{\hat{x} \in A^N}{\operatorname{argmin}} \| \hat{y} - R\hat{x} \|^2 \dots\dots (5)$$



Where  $\hat{y} = Q^T y$ ,  $\mathcal{J}$  is conjugate transpose,  $Q$  is an  $(N_r \times N_r)$  matrix has orthonormal columns (its columns are orthogonal unit vectors i.e.,  $Q^T Q = I_{N_r}$ ), and the matrix  $R$  is upper triangular[28]. In (5), the metric  $\|\hat{y} - R\hat{x}\|^2$  (cost function) is calculated depend on the square of the Euclidean distance or  $\ell^2$ -norm, which can be calculated iteratively as:

$$T_n = T_{n+1} + |c_n - r_{nn}\hat{x}_n|^2 \dots\dots\dots (6)$$

Where  $T_n$  is the partial Euclidean distance (PED) at the  $n$ -th layer,  $r_{ij}$  is the  $(i, j)$ -th component of  $R$ ,  $\hat{x}_i$  is the  $i$ -th component of  $\hat{x}$ ,  $c_n$  represents the interference cancellation, i.e.,

$$c_n = \hat{y}_n - \sum_{i=n+1}^N r_{ni}\hat{x}_i \dots\dots\dots (7)$$

where  $\hat{y}_n$  is the  $n$ th component of  $\hat{y}$ , and  $\hat{x}_i$  is the  $i$ th component of  $\hat{x}$ .

We can get the definitive metric  $T_1$  by calculating (6) iteratively, starting from  $T_{n+1} = 0$ . Until this moment, we are talked about the principle work of SD algorithm with Euclidean metric (referred to as  $\ell^2$ -norm SD), and now we will move on to talk about SD algorithm with Infinity norm (referred to as  $\ell^\infty$ -norm SD) in the next section.

### 2.1 SD algorithm with Infinity norm ( $\ell^\infty$ -norm SD)

As we have noted previously, the calculation in (6) demands the square operation of the  $\ell^2$ -norm, which is costly. To reduce this cost we will use the simplest norm known as Infinity-Norm or  $\ell^\infty$ -norm and the only alteration caused by adopting it in the detection process are that  $T_n$  in (6) is approximated as:

$$T_n = \max(T_{n+1}, |c_n - r_{nn}\hat{x}_n|) \dots\dots\dots (8)$$

From (8) can note there is no square operation and the non-decreasing property that appears in (6) is maintained. By checking out previous research to find out the effect of channel SNR on the complexity and the BER performance, we can say that the average number of visited nodes of the  $\ell^\infty$ -norm SD is less than that of the  $\ell^2$ -norm SD, in other words, the  $\ell^\infty$ -norm SD pruning more effectively than the SD with  $\ell^2$ -norm does. The BER performance of the  $\ell^2$ -norm SD is better from that of the  $\ell^\infty$ -norm SD in a small amount it can negligible.

The initial radius must be chosen carefully to ensure a fair comparison of the complexity between  $\ell^2$ -norm SD and  $\ell^\infty$ -norm SD. In our work and in this subsection, we use the approach proposed in [18, 29], as it shown below;

$$d_2^2 = \sigma_n^2 \Gamma_N^{-1} (1 - \epsilon) \dots\dots\dots (9),$$

$$d_\infty^2 = \sigma_n^2 \log(1 - \sqrt[N]{1 - \epsilon}) \dots\dots\dots (10),$$

Where  $d_2^2$  and  $d_\infty^2$  represent the initial radii of  $\ell^2$ -norm SD and  $\ell^\infty$ -norm SD respectively,  $1 - \epsilon$  is the probability of finding transmit vector inside the search sphere, and  $1 - \epsilon$  is set to a value close to 1, say,  $1 - \epsilon = 0.99$ [26].

### 2.2 Impact of initial radius on SD algorithm

SD algorithm was defined to execute MLD, which achieves complexity reduction by searching over the candidate points. SD algorithm depends on searching for the closest point, this search caused a problem, and the solving of this problem depends on the structure of the lattice. Therefore, the process requires estimation of an initial radius. The computational complexity of SD algorithm is controlled through the radius of the sphere. So, it is important to select an appropriate initial radius for SD algorithm to ensure a good performance and low complexity. The radius appears as a critical function in distinguishing the correct point in the lattice. Therefore, the initial radius chosen is still a problem in the detection process. If the radius is too small, that is mean that the probability of finding the closest point (ML solution) is low. If the radius is too large, that is mean that the sphere contains more candidate points, and thus lead to an increase in complexity [30]. In this subsection, the initial radius is selection as

$$d^2 = \gamma N \sigma_n^2 \dots\dots\dots (11),$$

where  $\gamma \geq 1$  is selected to guarantee the candidate point to captured [4].

Table 1 is the lookup table generated from (11) for the initial radius of SD algorithm for a 4x4 MIMO with 4-QAM and a given values of SNR according to different values of  $\gamma$ . According to (11) the initial radius can obtained by a constant value of gamma (gamma more than 1) multiplied by double number of receive antennas and the variance of noise. Therefore, in our MIMO system (4x4), the variance of noise is the main parameter that influence on the initial radius value, and  $SNR = 1/\sigma_n^2$ . So, we can see the relationship between initial radius values and SNR values in Table1.

From Table 1, we can observe that any specific radius can be obtained in any stage of implement the SD algorithm by looking up the table according to the SNR at any instant of time, and also can be observed increasing of the initial search radius in low SNR regime and vice versa, and this has a big impact on calculations of performance and complexity. From (11) can be concluded that the initial search radius increases with a number of receive antennas in MIMO system. The benefit from Table 1 appear when used it with genetic algorithms for enhancing detection process by training this algorithms on the values of the radii which gives better performance with less complexity.

**Table 1: The lookup table of initial radius and SNR**

SNR(dB)	$d1(\gamma = 2)$	$d2(\gamma = 5)$	$d3(\gamma = 8)$	$d4(\gamma = 10)$
1	32.0000	80.0000	128.0000	160.0000
2	25.4185	63.5463	101.6740	127.0925
3	20.1906	50.4766	80.7625	100.9532
4	16.0380	40.0950	64.1520	80.1900
5	12.7394	31.8486	50.9577	63.6971
6	10.1193	25.2982	40.4772	50.5964
7	8.0380	20.0951	32.1521	40.1902
8	6.3848	15.9621	25.5394	31.9242
9	5.0717	12.6791	20.2866	25.3583
10	4.0286	10.0714	16.1142	20.1428
11	3.2000	8.0000	12.8000	16.0000
12	2.5419	6.3546	10.1674	12.7093
13	2.0191	5.0477	8.0763	10.0953
14	1.6038	4.0095	6.4152	8.0190
15	1.2739	3.1849	5.0958	6.3697
16	1.0119	2.5298	4.0477	5.0596

### 3. Simulation Results

Using the system model for 4x4 uncoded MIMO with 4-QAM modulation, from Figure 1 we can discuss the use of the advantage  $\ell^\infty$ -norm rather than  $\ell^2$ -norm in SD algorithm, which show the BER performance of the SD algorithm. The comparison between  $\ell^\infty$ -norm SD,  $\ell^2$ -norm SD and MLD turns out that the performance of  $\ell^2$ -norm SD identical with the performance of MLD and there is a little degradation in performance of  $\ell^\infty$ -norm SD measuring with the performance of  $\ell^2$ -norm SD at a BER= $10^{-2}$ , about 0.2dB, is overlooked. The BER performance of the  $\ell^2$ -norm SD is always better than that of the  $\ell^\infty$ -norm SD, however, this difference difficult to distinguish when the SNR decreases.

Figure 2 depicts the relationship between the complexity of each of  $\ell^\infty$ -norm SD,  $\ell^2$ -norm SD and MLD, we can distinguish the difference in complexity between three curves, the complexity of the  $\ell^\infty$ -norm SD is less than that of the  $\ell^2$ -norm SD and MLD, and the complexity of the  $\ell^2$ -norm SD is less than that of the MLD. For example; the comparison between proposed  $\ell^\infty$ -norm SD and  $\ell^2$ -norm SD at minimum SNR=5 dB and maximum SNR=16 dB, the  $\ell^\infty$ -norm SD takes a time about (6.9 and 2.2) to search for nodes and the  $\ell^2$ -norm SD needs (5.9 and 2.07) to visit the nodes respectively. Therefore, the proposed  $\ell^\infty$ -norm SD needs 14.5% to 5.9% fewer complexities than  $\ell^2$ -norm SD.

Figure 3 depicts the relationship of BER performance and SNR for  $\ell^2$ -norm SD with different values of initial radius according to (11) with different values of gamma, we can note the good performance when use the large values of  $\gamma$  (8 and 10), the performance of this radii be the same as the performance of the ML, which represents large values for the initial radii as it shown in Table 1, and the performance degradation appear when the initial radii decrease with decreasing values of  $\gamma$  (2 to 5). For example at SNR=10.3 dB and a BER= $10^{-2}$  the two curves of ML and SD with  $d3(\gamma=8)$  identical, this matching make the initial radius of a SD increasing from  $d1(\gamma=2)$  to  $d3(\gamma=8)$  to obtain for a better performance. In other words, the large initial radius condition implies that there are many candidate points within the sphere, the BER



performance will be a good. If the initial radius is small, that means there is no lattice point within the sphere which leads to repetitive search and hence, degraded the performance.

Figure 4 show the complexity and its relationship with initial radii, which is reverse the state of the performance that were exhibited in Figure 3. The complexity is high when using a large initial radius due to a large number of candidate points within the sphere. Hence, the complexity cannot be reduced effectively. The complexity is low when using a small initial radius due to the small number of candidate points within the sphere, but the initial radii must not be very small because this lead to there is no candidate point within the sphere and this leads to an iterative search and hence, increase the complexity.

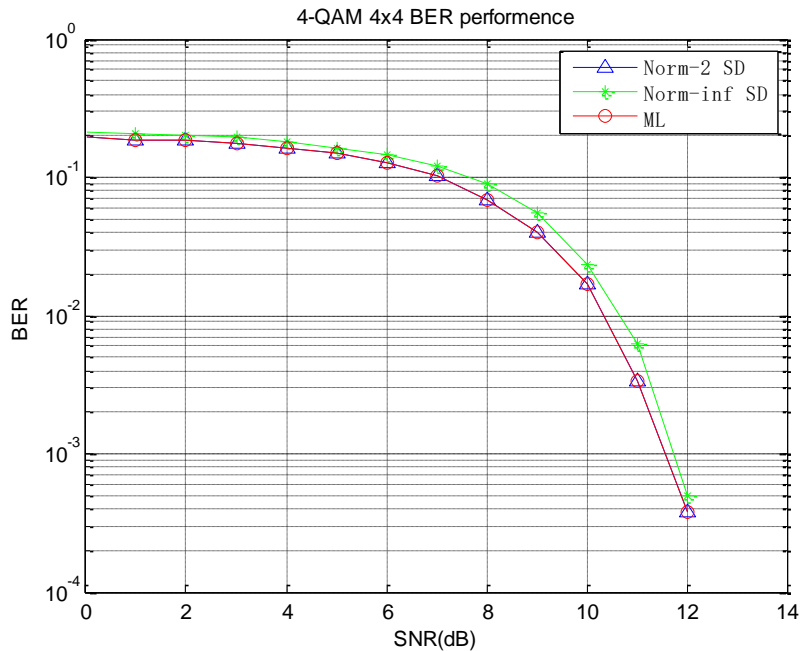


Fig 1: BER performances of the  $\ell^2$ -norm SD,  $\ell^\infty$ -norm SD and MLD for 4x4 MIMO with 4-QAM.

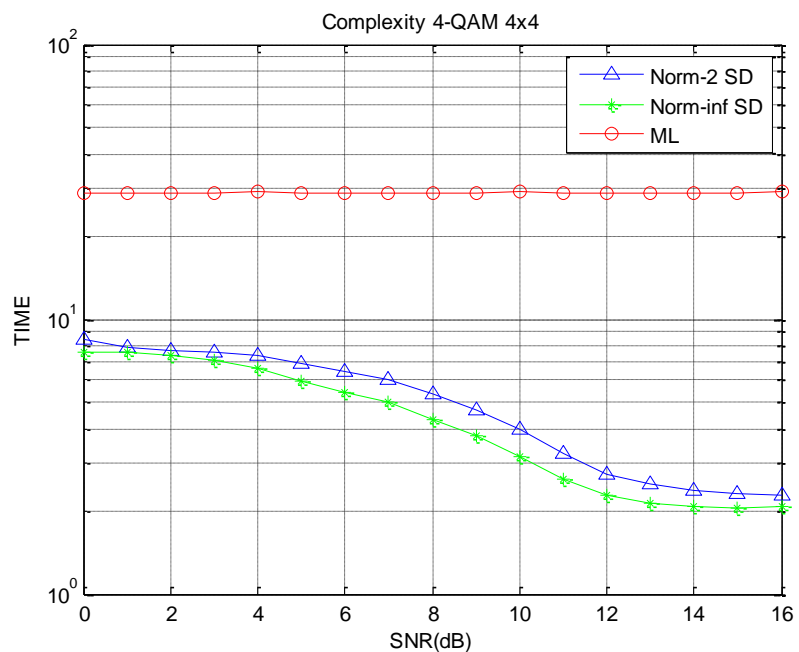


Fig 2: Complexity of the  $\ell^2$ -norm SD,  $\ell^\infty$ -norm SD and MLD for 4x4 MIMO with 4-QAM.

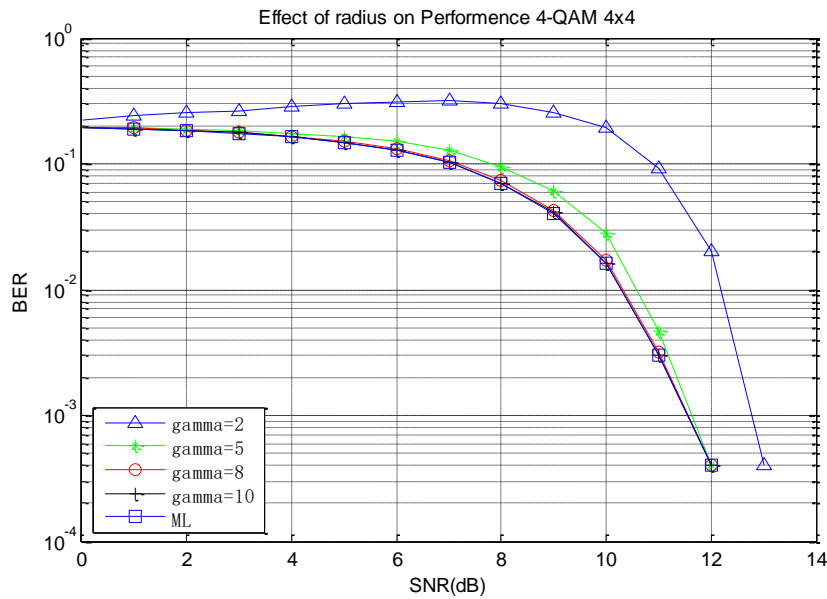


Fig 3: BER vs. SNR for  $\ell^2$ -norm SD with different values of initial radii for 4x4 MIMO with 4-QAM.

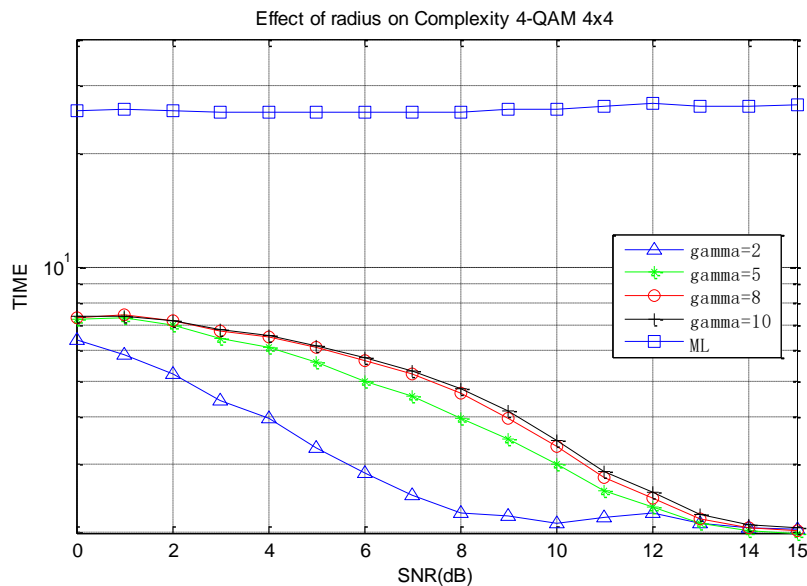


Fig 4: Complexity of  $\ell^2$ -norm SD with different values of initial radii for 4x4 MIMO with 4-QAM.

#### 4. Conclusions

We applied sphere decoding (SD) based on the  $\ell^\infty$ -norm instead of SD based  $\ell^2$ -norm and we found through simulation results that the degradation in BER performance is very small and can be negligible when to use the  $\ell^\infty$ -norm. In the other side, the issue of complexity appears an enhancement between the two curves of  $\ell^\infty$ -norm SD and  $\ell^2$ -norm SD in spite of being a small to favor  $\ell^\infty$ -norm SD. You can add this improvement to other advantages of  $\ell^\infty$ -norm SD that it has been mentioned previously. Thus, the effectiveness of the  $\ell^\infty$ -norm SD calculation can appear with a straightforward implementation of MIMO symbol detection as a valid alternative for practical hardware implementation in the future wireless communication systems. In terms of the impact of initial search radius in the BER performance and complexity for SD algorithm, the simulation results proved that the BER performance of the SD algorithm tends to be equal to the BER performance of MLD when the search radius increases. The complexity calculation is influenced by choosing the initial search radius, therefore, the complexity becomes high when using a large initial radius, and its low when using a small initial radius, but not very small to avoid the iterative search and hence, increase the complexity again.



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