

# Fuzzy Weighted Ordered Weighted Average-Gaussian Mixture Model for Feature Reduction

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## ABSTRACT

*Feature reduction finds the optimal feature subset using machine learning techniques and evaluation criteria. Some of the irrelevant features are existed in the real-world datasets that should be removed by using the multi criterion decision approach. The relevant features are determined by using the WOWA criteria in fuzzy set. There are two important criteria are considered such as preferential weights and importance weights of features. These weights are used to find the irrelevant features and they are removed from the mixture. In this context, WOWA operator has the capability of assigning the preferential weights and important weights to the features. It helps to obtain the irrelevant, by selecting the relevant features using the weights in the feature reduction process. The objective of this paper is to propose a FWOVA approach helps to discard the irrelevant features by avoiding the overfitting and improve the accuracy of the cluster. The irrelevant features are determined by applying WOWA. By applying WOWA, the irrelevant features are examined and it is removed from the Gaussian Mixture using (RPEM).*

## Indexing terms

Gaussian Mixture Model, OWA, WOWA, Feature Reduction.

## Academic Discipline and Sub-Disciplines

Computer Science

## SUBJECT CLASSIFICATION

Data Mining, Feature Reduction

## 1. Introduction

Feature reduction is a kind of feature selection problem, which focuses on relevant features selection from the feature space. The features are recognized and evaluated by the evaluation criterion. Each feature is identified by using criterion value, which is accepted for the data mining process. There are relevant, irrelevant and insignificant features are existing in the real world dataset. The relevant features are determined by using the weighting and ranking methods.

The relevance depends on nature of the feature and it's having relationship with class variable. The relevance is one of the criteria for decision making. Only relevant features are selected as the objective of the feature reduction but irrelevant feature is also selected in this process. So, there is a mechanism called a multi criteria decision making in Fuzzy Set, that is used for finding the irrelevant features by selecting the relevant features using relevance and reliability. This is achieved through Fuzzy Weighted Ordered Weighted Average (FWOWA) approach for identifying the reliability and relevance of the features in the dataset.

Through this approach, the relevant features are identified and irrelevant features are eradicated in the reduction process. So, highly relevant features are selected and their relevance and reliability are high. This problem is solved through one of the fuzzy weighing techniques, namely Weighted Ordered Weighted Average (WOWA). This weighing approach is used to select the relevant features and ranked based on the weights of the features during the reduction process. After the reduction, the winning component mixtures are evaluated by Rival Penalization Mechanism. These mixtures are used to form the clustering using GMM.

## 2. Motivation

Kim et.al [19] presented a feature reduction technique for reducing the features in clusters with guidance of the k-means. The aim of this technique was to remove the irrelevant information and the learning performance was improved by reducing the storage cost of database using this approach.

Martin H. C. Law et al. [20] introduced a feature reduction technique. The important issue in clustering is selection of features and the finding of irrelevant features. MML selection criterion was used for selecting the relevant features in the feature space. MML criterion identified the irrelevant features by applying the Expectation Maximization.

Yuanhong Li et.al [21] proposed a new feature reduction technique for selecting the feature by removing the irrelevant features. In this work, the different relevant feature subsets were identified for different clusters, which were usually smaller than the globally relevant feature and can be irrelevant to some clusters. In this experiment, locally

relevant features might be treated as globally irrelevant. These features were hindering the clustering performance. MML Criterion was used to remove the irrelevant features from the locally relevant features.

Hong Zeng and Yiu-ming Cheung [22] proposed a novel feature reduction method for selecting the relevant features from the localized clusters. The selected features were useful for local cluster presentation but they were not useful for global cluster structure presentation. It estimated the parameters of each component in a mixture based on the observations. In this experiment, scoring criterion was used to identify the relevant features. The selected relevant features useful for global cluster structure is one of the issues.

Hong Zeng et al. [23] introduced a new feature selection technique using RPEM, which was used to obtain the optimal number of features from the feature space. Scoring Criterion and Markov Blanket filtering were adopted to obtain the optimal number of features. This technique besides selecting the relevant features, removed the redundant features, but it has left out some irrelevant from the feature space.

This study shows the existence of irrelevant features after finding the relevant features from the feature space. This issue is addressed by using the weighted Ordered Weighted Average (WOWA) aggregation. WOWA aggregation is used to order the weights based on the relevance of the features and reliability of data values. The preferential and importance weights are assigned to features from minimum, mean and maximum. These weights are used for finding the relevant features and removing the irrelevant features left out from the feature space using Orness criterion in Fuzzy set.

## Preliminaries

In this section, the preliminaries of the FOWWA approach are discussed for the reducing the features by removing irrelevant features using WOWA operator.

### 3.1 Significance of WOWA

A simple arithmetic mean was used to perform rough compensation between high and low values. In this scenario, the weighted mean was applied to identify the various compensation values of dataset [3]. The Ordered Weighted Average Operator was compared with Weighed mean, in the form of Behavior Analysis [4] [1] [2]. Weighted Mean (WM) aggregated trust values from different sources. It was considered as the reliability of the source (Importance Weight). OWA operator aggregates the weighted trust values based on their size, without considering the sources (Preference Weight) [4]. The Preferential weight was introduced in fuzzy optimization and it was called Ordered Weighted Averaging [2]. In OWA Aggregation, the weights were obtained as ordered values rather than to the specific criteria. This OWA aggregation technique was applied in different decision making problems [5] [6] [7]. The OWA optimization with monotonic weights was used in standard linear program of higher dimension [8]. The OWA operator was applied to model various aggregation functions from the highest through the arithmetic mean to the lowest. The importance weights were not calculated assigned by OWA aggregation technique and Weighted Mean also cannot be expressed in OWA [11]. The importance weight was incorporated with Ordered Weighted Average, which was called Weighted Ordered Weighted Average (WOWA). This average becomes the weighted mean in the case of equaling all the preference weights and OWA averages also are reduced for equaling all the importance weights [10]. In this context, the relevant features are identified using WOWA by eradicating the irrelevant features in the sleep apnea dataset [13]. In WOWA aggregation and Principal Components Analysis, OWA operator was used to identify the relevant features and reliability of the variable was identified using WOWA. Through this irrelevant features are removed in the dataset [9]. In Gaussian Mixture feature reduction, the irrelevant features were identified in the observations and it affects the computational complexity in clustering process due to the curse of dimensionality [12].

WOWA aggregation method is used to bias the data, with respect to relevance and reliability. It correlates the input features in order to reduce the dimensionality in one or more factors, OWA weight the features, and WOWA weight both the features and the data values. Weighted mean (WM) is an aggregation technique incorporated in WOWA which has as input a data vector and a weight vector. The weight vector contains one degree of reliability value between 0 and 1, for each corresponding feature.

### 3.2 The WOWA Operator

The WOWA aggregation was used to order the Order Weighted Averaging (OWA) for finding the preferential weights and model various preferences with respect to the risk. The importance weights are also calculated for the reliability of the data values in the Dataset [14]. The advantage of using the WOWA is that it unifies the WA and the OWA taking into account the degree of importance in Multi Criteria decision Making [15].

The orness of the WOWA operator was used to know the importance of the sources and the importance of the values. This technique was used to combine information by two ways (i) corresponding to the same attribute that come either from different sources or from the single source (ii) corresponding to different attributes [16]. The WA and OWA operators were used to unify the WOWA formulation, which identify the importance and preferential weights of data values in the dataset [17] [18].

The Weighted OWA operator is an efficient approach, because it combines the OWA operator and Weighted Mean. In WOWA, two sets of weights are used namely,  $p$  is considered as relevance of the sources and  $w$  considered as relevance of values.

**Definition**

Let  $p$  and  $w$  be two weight vectors of dimension  $n$

i)  $p = [p_1, p_2 \dots p_n]$  where  $p_i \in [0,1], \sum_i p_i=1$

ii)  $w = [w_1, w_2 \dots w_n]$  where  $w_i \in [0,1], \sum_i w_i=1$

A mapping  $f_{WOWA} : R^n \rightarrow R$  is a Weighted Ordered Weighted Averaging operator of dimension  $n$ .

$$f_{WOWA} (a_1 , a_2 \dots a_n) = \sum_i \omega_i a_{\sigma(i)} \tag{1}$$

Where  $\{\sigma(1), \sigma(2) \dots \sigma(n)\}$  is a permutation of  $n$ , such that  $a_{\sigma(i-1)} \geq a_{\sigma(i)}$   $i = 2..n$  and weight  $\omega_i$  is defined as

$$\omega_i = w^* \left( \sum_{j \leq i} p_{\sigma(j)} \right) - w^* \left( \sum_{j < i} p_{\sigma(j)} \right) \tag{2}$$

With  $w^*$  is a monotonic function that interpolates the points  $(\frac{1}{n}, \sum_{j \leq i} w_j)$  together with the point  $(0,0)$ . Term  $\omega$  denotes the set of weights  $\{\omega_i\}$  i.e  $\omega = \{\omega_1, \omega_2 \dots \omega_n\}$

Given  $p$  and  $w$  and a data vector  $a$ . Let  $S = \{(\frac{1}{n}, \sum_{j \leq i} w_j) \mid i = 1 \dots n\} \cup \{0, 0\}$ . One needs to define the interpolation function  $w^*$  interpolating  $S$ . Two possible approaches are applied in the interpolation function. 1) Define a vector  $w$  and then function  $w^*$  is determined 2) Define function  $w^*$ . In the first approach, the monotonic function  $w^*$  is obtained by applying any method, starting from monotonic data points in the unit interval. In the second approach, the set of weights  $\omega$  is derived from  $w^*$ , where  $w^*$  is monotonically increasing function,  $w^* \in [0, 1]$  interval with  $w^*(0) = 0$  and  $w^*(1) = 1$ .

The  $w$  is composed of values of the  $w_i = \frac{k}{n}$  and the following two approaches are considered for normalizing the vector. The two different approaches are used for finding the relevance and reliability to the data values. . It helps to avoid overfitting of the model by removing irrelevant features. *Diffident Approach:* The WOWA operator is employed for permutation of data values  $a_i$  (where  $a_{\sigma(i-1)} \geq a_{\sigma(i)} \forall i = 2 \dots n$ ). The weights  $w_i = \frac{k}{n}$ , where  $k = 1, 2 \dots n$  have to be normalized by dividing them by their sum.  $w_i = \frac{\frac{k}{n}}{\sum_{k=1}^n \frac{k}{n}}$ . *Confident Approach:* By choosing a function interpolating the points

$(\frac{i}{n}, \sum_{j \leq i} w_j)$ , where  $w_i = \frac{k}{n}$ ,  $k = n, n-1 \dots 1$ . The weights are normalized based on these approaches for finding the relevant and reliable features.

**4 FOWA Algorithm**

**FOWA Algorithm**

**Procedure Feature Reduction (F,β,γ,T)**

**Step 1 :**  $F : X \rightarrow [0, 1]$

$$F = \mu_1/x_1 + \dots + \mu_n/x_n$$

**Step 2:** **Calculate Weights:  $F_r \in F$**

$$W' = \sum_{i=1}^n \omega_i a_{\sigma(i)}$$

**Step 3:** ***\* Feature reduction and Filtering\****

$$R' \leftarrow F - (F_r \mid \text{Rank}_r < \beta, F_r \in F)$$

$$R'' = [(U_{wi} - L_{wi}) + (M_{wi} - L_{wi})] / n + L_{wi}$$

**Step 4:**  $R'' \leftarrow$  Projection of  $R$

**Step 5:**  $R \leftarrow R''$

**Objective Function**

RPEM algorithm finds the preferential and importance weights and mixture by applying the following equation.

$$\text{orness}(w) = \sum_{i=1}^n \frac{(n-i)}{n-1} * w_i \tag{3}$$

The algorithm can be modelled as mathematical

$$\begin{aligned} \text{Maximize orness } (w) &= \sum_{i=1}^n \frac{(n-i)}{n-1} * w_i \\ \text{Subject to } \text{wowa} &= \sum_{i=1}^n w_i a_{\sigma(i)} = O \quad 0 \leq O \leq 1 \\ w_i &= w_1 + \dots + w_n = 1, \quad 0 \leq w_i, i=1 \dots n \end{aligned}$$

## 5 Experimental Results

In this experiment, four datasets are used to identify the various relevant features by removing the irrelevant features using FWOWA algorithm.

### 5.1 Analysis with Wine Dataset

In this experiment, the OWA weights are used in this analysis. It is useful for identifying the preferential weights of the features. Another weighing technique is also used for identifying the importance of the features in the dataset, namely WM, based on the data values. According to WOWA, Both Preferential (p) and Importance Weights are identified according to the preferential of the features.

**Table 1. OWA weights of Wine Dataset**

$\mu(x)$	f1	f2	f3	f4	f5	f6	f7	f8	f9	f10	f11	f12	f13	$\Sigma$
0	0	0	0	0	0	0	0	0	0	0	0	0	1	1
0.1	0	0	0.06	0.013	0	0.069	0	0	0	0	0	0	0.858	1
0.2	0	0	0.041	0	0	0.284	0	0	0	0	0	0	0.675	1
0.3	0	0	0.014	0	0	0.494	0	0	0	0	0	0	0.492	1
0.4	0	0	0	0	0	0.686	0	0	0	0	0	0	0.314	1
0.5	0	0	0	0	0	0.857	0	0	0	0	0	0	0.143	1
0.6	0	0	0	0	0.18	0.8	0.02	0	0	0	0	0	0	1
0.7	0	0	0.2	0	0.8	0	0	0	0	0	0	0	0	1
0.8	0	0	0.8	0	0.2	0	0	0	0	0	0	0	0	1
0.9	0	0.8	0.2	0	0	0	0	0	0	0	0	0	0	1
1	1	0	0	0	0	0	0	0	0	0	0	0	0	1

Table 1 showst the preferential weight of Wine dataset features. Each feature is weighed (p) based on relevance. For instance, vector  $p = [1, 0.8, 0.8, 0.013, 0, 0, 0, 0, 0, 0, 1]$  indicates that extreme values are reliable and other values are not reliable. Here the weights are ordered based on the criterion value using Orness. The relevance of features is determined by applying preferential weights.

**Table.2. Various Criterion Values of Features**

Criterion	C1	C2	C3
F1	1	1	1
F2	0.8	0.8	0.8
F3	0.8	0.8	0.6
F4	0.008	0.13	0.4
F5	0.8	0.8	0.6
F6	0.857	0.857	0.8
F7	0.029	0.020	0.713
F13	1	1	1

Table 2 shows that the feature weights are calculated based on the relevance of the values; it is also called reliability. Each feature has its own importance, which is measured based on reliability of ordered values of each feature and varies for each feature. It divulges how the data values are reliable and which is useful for the prediction of the reliable feature in the dataset. The weighting vector  $w$  is determined according to the importance to the data values.

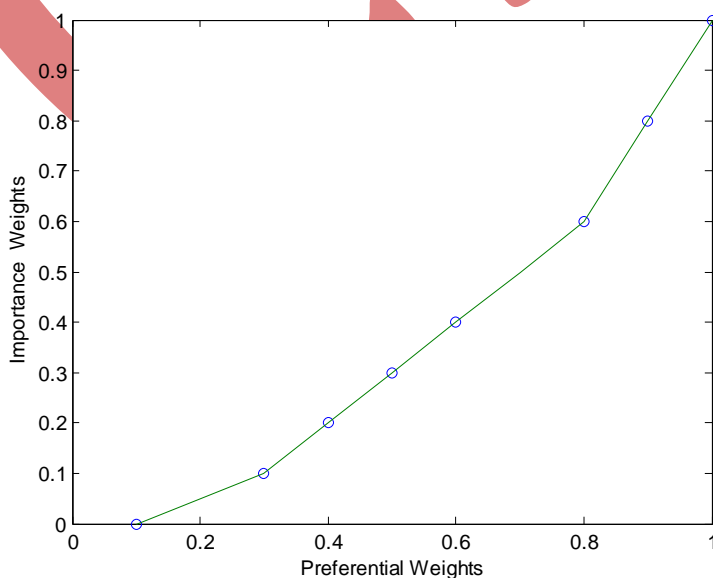
The vector  $w$  is obtained from the reliability depending on the information source. For Instance  $w = [0.142, 0.142, 0.142, 0.006, 0.142, 0.142, 0.142, 0.142]$ , the highest weighted values are more reliable, which are matched with preferential weighting vector  $w$ . The monotonic function is applied and the set of weights are derived from  $w$ , where  $w$  is monotonically increasing function within the  $[0, 1]$  interval with  $w(0) = 0$  and  $w(1) = 1$ .

By using the preferential ( $p$ ) and importance ( $w$ ) weights, the weights ( $w'$ ) are derived. The derived weights ( $w'$ ) are utilized for finding the relevant features and the data values. Hence, the weights ( $w'$ ) are composed of values in the form of normalized vector in Table.3.

**Table 3 Normalized Vector Values of Wine Dataset (Diffident)**

Importance Weight	Preferential Weight
0.1	0.0
0.3	0.1
0.4	0.2
0.5	0.3
0.6	0.4
0.8	0.6
0.9	0.8
1.0	1.0

The calculated weights, such as preferential and importance weights are used to draw the interpolating function by applying diffident approach. Figure.2 shows the normalized weights of both preferential (x-axis) and importance (y-axis) weights of the wine dataset. It divulges that the irrelevant features exist in wine dataset. It affects the accuracy of the GMM Clustering

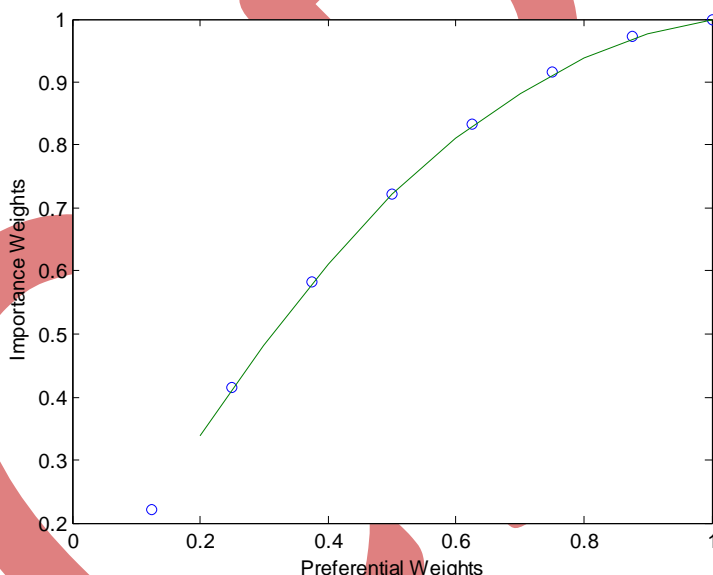


**Figure 1. Interpolation function  $w'$  diffident approach**

For confident approach, highly relevant and reliable weighted features are derived from the feature space based on importance and preference weights and it is tabulated in Table 4. The weights are normalized and they are used to identify the highly relevant and reliable feature for forming the Component Mixture.

**Table 4. Normalized Vector Values of Wine Dataset**

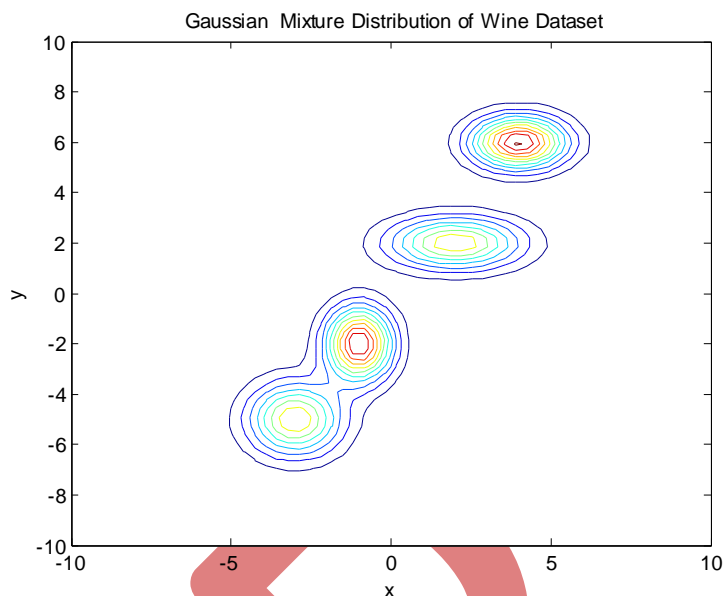
Importance Weight	Preferential Weight
0.125	0.222
0.25	0.416
0.375	0.583
0.5	0.722
0.625	0.833
0.75	0.916
0.875	0.972
1	1



**Figure 2. Interpolation function w\*confident approach**

The calculated weights, such as preferential and importance weights are used to draw the interpolating function by applying confident approach. Figure.3 shows the normalized weights of both preferential (x-axis) and importance (y-axis) weights of wine dataset. It divulges that the relevant features are used for clustering, so that it increases the accuracy of GMM Clustering. Diffident and Confident approaches are applied to evaluate the dataset and find the relevance and reliability of features.

In different epochs, the relevant features are identified using preferential weight and importance weight using Orness criterion. In each epoch, the various features are selected and they are ordered from maximum to minimum values. The maximum weighted features are used to form winning component mixture and less weighted features are removed from the mixture. Each component mixture is evaluated in two ways such as by model and sampling.



**Figure 4. Gaussian Mixture Distribution of Wine Dataset**

In Figure.4, Component mixtures with highly relevant and reliable features are diagrammatically represented. Here, both Preferential and Importance Weighted features are selected and component mixtures are formed using observed features. Each data points are appended with independent variables and are sampled from a standard normal distribution. By applying FWOWA, the component mixtures are refined and cluster structures are formed with the help of data points selected by the algorithm. RPEM algorithm has accurately silhouetted the data points by using Gaussian Mixture Model structures and component parameters are estimated.

### 5.2 Analysis with Ionosphere Dataset

In Ionosphere dataset, there are 351 data points with two class variables in it. According to WOWA, the weighting vector  $w$  is identified using OWA preferential weights.

**Table .5. OWA weights of Ionosphere Dataset**

$\mu(x_0)$	f1	f2	f18	f20	f22	f24	f27	f34	$\Sigma$
0	0	0	0	0	0	0	0	1	1
0.1	0	0	0	0	0	0.068	0.432	0.5	1
0.2	0	0	0	0	0.308	0	0.501	0.19	1
0.3	0	0	0	0.312	0.283	0	0.405	0	1
0.4	0	0	0.132	0.68	0.17	0	0.018	0	1
0.5	0.101	0.035	0.229	0.635	0	0	0	0	1
0.6	0.076	0.249	0.138	0.537	0	0	0	0	1
0.7	0.052	0.463	0.046	0.439	0	0	0	0	1
0.8	0.036	0.662	0	0.302	0	0	0	0	1
0.9	0.03	0.847	0	0.124	0	0	0	0	1
1	1	0	0	0	0	0	0	0	1

Table 5 shows the preferential weights of Ionosphere dataset features. Each feature is weighed ( $p$ ) based on relevance using OWA operators. For instance, weighting vector  $p = [1, 0.847, 0.941, 0.822, 0.351, 0.588, 0.117, 0.501, 0.165, 0.437, 1]$  indicates that the largest values are reliable. The weights are ordered based on the criterion value using Orness.



**Table.6. Various Criterion Values of Features**

Criterion	C1	C2	C3
F1	1	1	1
F2	0.847	0.863	0.863
F18	0.941	0.933	0.928
F20	0.68	0.822	0.328
F22	0.308	0.261	0.351
F25	0.588	0.117	0.261
F26	0.114	0.117	0.038
F27	0.501	0.169	0.405
F28	0.063	0.165	0.127
F30	0.231	0.437	0.205
F34	1	1	1

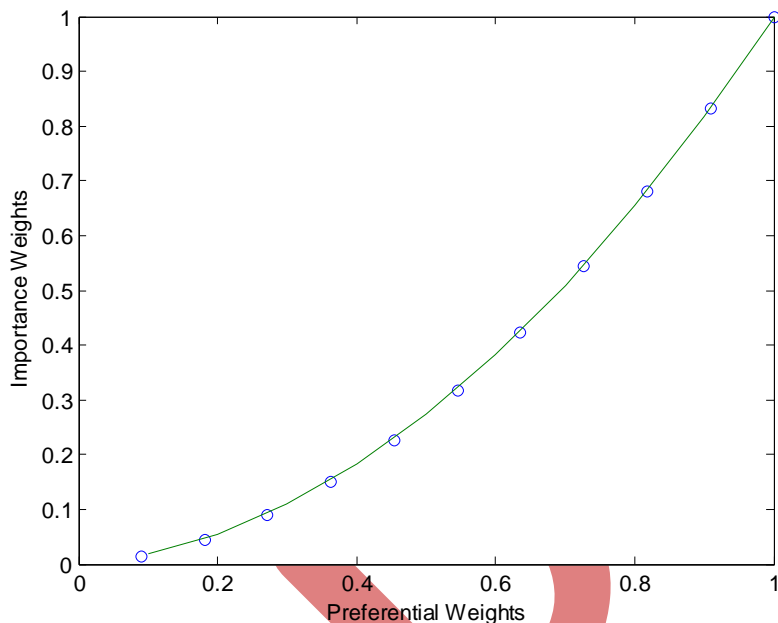
Table.6 shows the calculated feature weights based on the relevance. Each feature has its own importance; it divulges how the data values are reliable. It helps to identify the reliable feature for prediction. The weighting vector ( $w$ ) is determined according to the importance to the data values in different epochs. For Instance, vector  $w = [0.16, 0.16, 0.16, 0.16, 0.033, 0.033, 0.033, 0.033, 0.033, 0.033, 0.033, 0.16]$  indicates that the highest values are relevant.

The preferential ( $p$ ) and importance ( $w$ ) weights are used for deriving the weights ( $w'$ ), for finding the relevant features and data values. The weights ( $w'$ ) are mapped into normalized vector in Table.7.

**Table 7. Normalized Weight Vector Values of Ionosphere Dataset (Diffident)**

Importance Weight	Preferential Weight
0.090	0.015
0.181	0.045
0.272	0.090
0.363	0.151
0.454	0.227
0.545	0.318
0.636	0.424
0.727	0.545
0.818	0.681
0.909	0.833
1	1





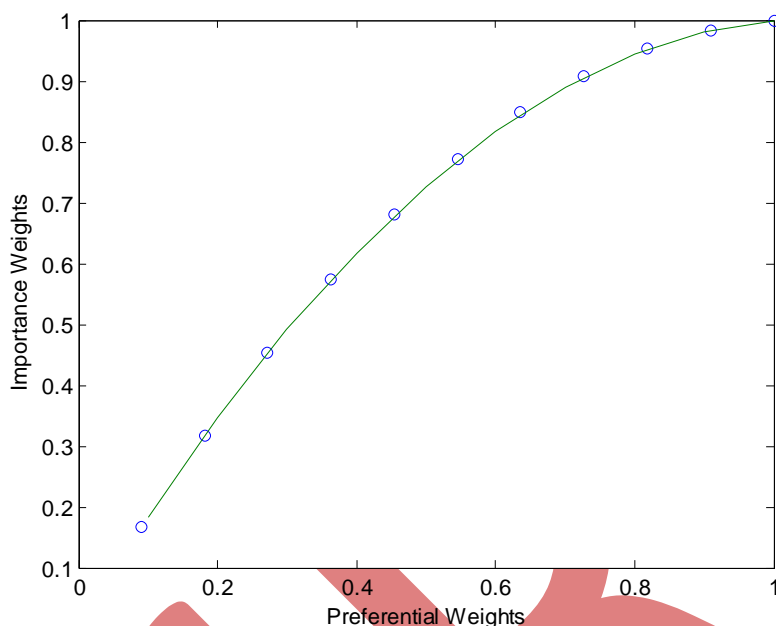
**Figure 5. Interpolation function w\* diffident approach**

The calculated preferential and importance weights are used to present the interpolating function by applying diffident approach. Figure 5 shows the normalized weights of both preferential and importance weights of the Ionosphere dataset. It reveals that the irrelevant feature affects the accuracy of clustering process.

For the confident approach, the highly relevant features are selected and identified by applying interpolation function. They are mapped as a normalized weight vector listed in the Table 8.

**Table 8. Normalized Vector Values of Ionosphere Dataset (Confident)**

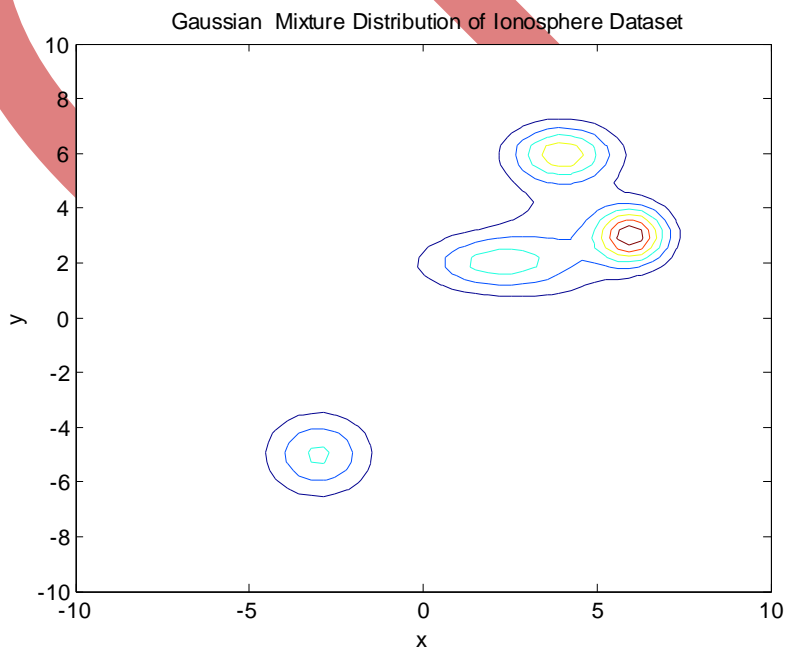
Importance Weight	Preferential Weight
0.090	0.166
0.181	0.318
0.272	0.454
0.363	0.575
0.454	0.681
0.545	0.772
0.636	0.848
0.727	0.909
0.818	0.954
0.909	0.984
1	1



**Figure 6. Interpolation function  $w^*$ confident approach**

The preferential and importance weights are used to draw the interpolating function by applying confident approach. Figure .6 shows the normalized weights of both preferential and importance weights of the lonosphere dataset. It reveals that the highly relevant features exist in the dataset, which are used for clustering process. The feature weights are evaluated by applying the diffident and confident approach.

In different epochs, the ornness criterion was used to identify the relevant features using preferential weight and importance weight. In each epoch, the various features are selected and their weights are ordered. The Winning Component Mixtures are formed and less weighted features are removed.



**Figure 7. Gaussian Mixture Components of lonosphere**

In Figure 7, Component mixtures with highly relevant and reliable features are diagrammatically represented. Each data points are appended with independent variables and are sampled from a standard normal distribution. By using FWOWA, the component mixtures and cluster structures are formed. Various clusters are silhouetted using Gaussian Mixture Model structures and component parameters are estimated.

### 5.3 Analysis with Wdbc Dataset

In **Wdbc** dataset, there are 351 data points with two class variables in it. The weighting vector **w** is identified using OWA preferential weights.

**Table 9. OWA weights of Wdbc Dataset**

$\mu(x)$	F4	F5	F7	F8	F9	F10	F11	F12	F13	F14	F15	F16	F18	$\Sigma$
0	0	0	0	0	0	0	0	0	0	0	0	0	1	1
0.1	0	0	0	0	0	0	0	0	0	0	0	0.85	0.15	1
0.2	0	0	0	0	0	0	0	0	0	0.4	0.6	0	0	1
0.3	0	0	0	0	0	0	0	0.1	0.9	0	0	0	0	1
0.4	0	0	0	0	0	0	0.9	0	0.1	0	0	0	0	1
0.5	0	0	0	0	0.5	0.5	0	0	0	0	0	0	0	1
0.6	0	0	0.2	0.8	0	0	0	0	0	0	0	0	0	1
0.7	0	0.45	0.55	0	0	0	0	0	0	0	0	0	0	1
0.8	0.6	0.4	0	0	0	0	0	0	0	0	0	0	0	1

Table 9 shows the preferential weight of features. Each feature is weighed based on the relevance using ordered weighing approach. For instance, weighting vector  $\mathbf{p} = [0.6, 0.45, 0.55, 0.8, 0.5, 0.5, 0.9, 0.4, 0.6, 0.85, 1]$  indicates that the largest values are the most important ones. Here the weights are ordered based on the criterion value. The weights are calculated based on the membership values and features are determined.

**Table10. Various Criterion Values of Features**

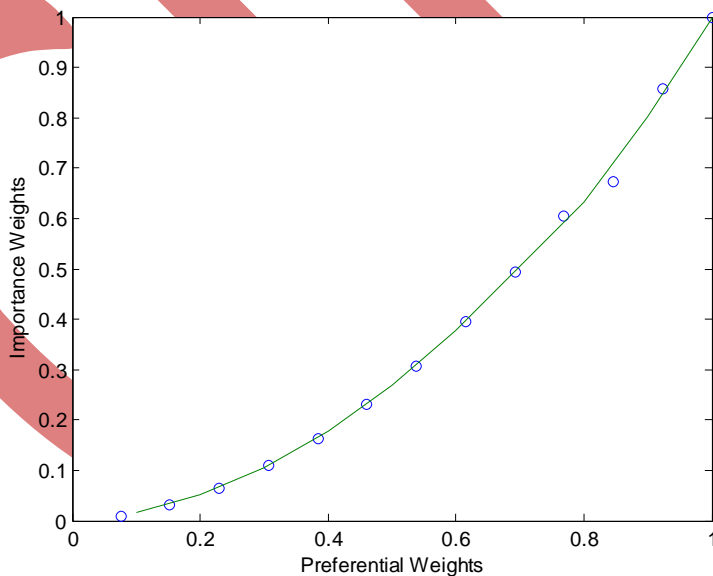
Criterion	C1	C2	C3
F4	0.6	0.7	0.95
F5	0.45	0.491	0
F7	0.55	0.55	0.8
F8	0.8	0.8	0
F9	0.5	0.5	0
F10	0.5	0.5	0
F11	0.9	0.5	0.4
F12	0.1	0	0.6
F13	0.9	0	0
F14	0.4	0.7	0.5
F15	0.6	0	0.5
F16	0.85	0.85	0
F18	0.15	0.15	0.4

Table10 shows the feature weights based on the reliability of values. It reveals how the data values are reliable and relevant. The weighting vector (**w**) is determined in different epochs and different criterion. For Instance, vector  $\mathbf{w} = [0.11, 0.03, 0.11, 0.11, 0.03, 0.03, 0.11, 0.03, 0.11, 0.11, 0.03, 0.11, 0.03]$  indicates that the highest values are reliable.

The highest weighted values (**w**) are more reliable, which are matched with preferential weighting (**p**). The set of weights (**w'**) is derived from preferential and importance weight. The weights (**w'**) are normalized by using diffident and confident approach.

**Table11. Normalized Vector Values of**

Importance Weights	Preferential Weights
0.076	0.010
0.153	0.032
0.230	0.065
0.307	0.109
0.384	0.164
0.461	0.230
0.538	0.307
0.615	0.395
0.692	0.494
0.769	0.604
0.846	0.673
0.923	0.857
1	1

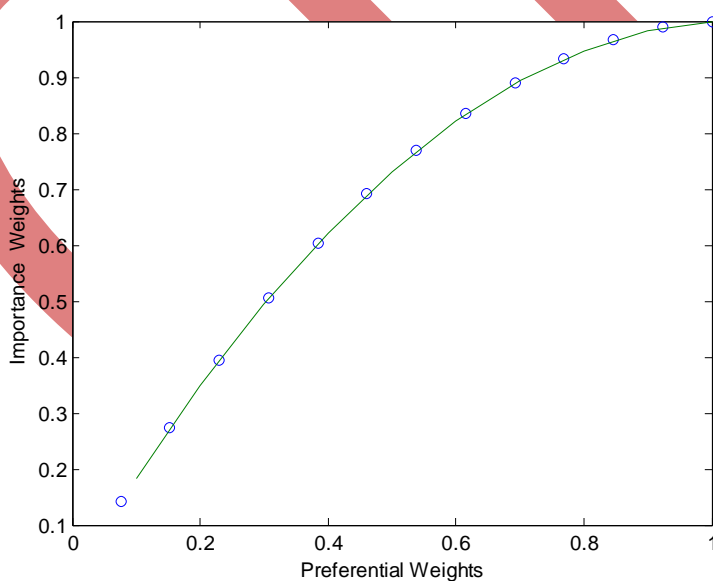


**Figure 8. Interpolation function  $w^*$  diffident approach**

Figure 8 shows the normalized weights of both preferential and importance weights of the wdbc dataset. The irrelevant features affect the accuracy of clustering process. The relevance and reliable values in the weight vectors are used as normalized vector for interpolating the data points in confident approach.

**Table 12 Normalized Vector Values of Importance and Preferential Weights**

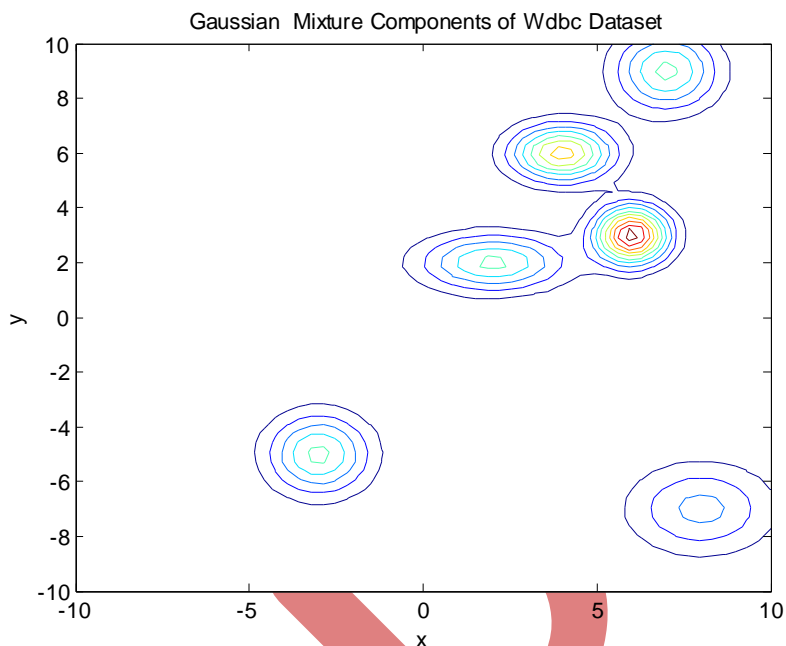
Importance Weights	Preferential Weights
0.076	0.142
0.153	0.274
0.230	0.395
0.307	0.505
0.384	0.604
0.461	0.692
0.538	0.769
0.615	0.835
0.692	0.890
0.769	0.934
0.846	0.967
0.923	0.989
1	1



**Figure .9. Interpolation function w\*confident approach**

Figure 9 shows the normalized weights of both preferential and importance weights of the Wdbc dataset. It reveals that the highly relevant and reliable features are obtained for clustering. Diffident and confident approaches are employed to evaluate the feature weights.

Based on the orness criterion, the relevant features are determined using the preferential weight and importance weight. In several epochs, various features are selected and their weights are ordered using threshold. The Winning Component Mixtures are formed and their similarity value is high.



**Figure10. Gaussian Mixture Components of Wdbc Dataset**

In Figure 10, the highly relevant and reliable features are diagrammatically represented in the form of component mixture. Each data points are composed of mixture with independent variables and are sampled from a standard normal distribution. The component mixtures and cluster structures are formed with the help of data points. FWOVA algorithm clusters the various data points by using Gaussian Mixture Model structures. The new clusters structures are formed using FWOVA and error rate index is also calculated for accuracy.

**5.4 Analysis with Sonar Dataset**

In Sonar dataset, data points with two class variables exist. In WOVA, two kinds of weights are used for finding the relevant and reliable features.

**Table13. OWA weights of Sonar Dataset**

$\mu(x)$	F1	F3	F13	F15	F18	F19	F20	F21	F22	F23	F40	F45	$\Sigma$
0	0	0	0	0	0	0	0	0	0	0	0	1	1
0.1	0.1	0	0	0	0	0	0	0	0	0	0	0.9	1
0.2	0.101	0	0	0	0	0.167	0	0	0	0	0	0.732	1
0.3	0	0	0	0	0	0.09	0	0.227	0.235	0	0	0.448	1
0.4	0	0	0	0	0	0	0	0	0.619	0.085	0.295	0	1
0.5	0	0	0	0	0	0	0	0	0	1	0	0	1
0.6	0	0	0	0	0.7	0	0.3	0	0	0	0	0	1
0.7	0	0	0.4	0.6	0	0	0	0	0	0	0	0	1
0.8	0	0.32	0.68	0	0	0	0	0	0	0	0	0	1
0.9	0.197	0.524	0.279	0	0	0	0	0	0	0	0	0	1
1	1	0	0	0	0	0	0	0	0	0	0	0	1

Table13 shows the preferential weight of sonar features. The features are weighed based on the information source using OWA. For instance, weighting vector  $p = [1, 0.524, 0.68, 0.6, 0.7, 0.167, 0.3, 0.227, 0.619, 0.085, 0.295, 1]$  indicates that the largest values are the preferential features. Here the weights are ordered based on the criterion value. Various weights are calculated for each feature based on the membership values.

**Table 14. Various Criterion Values of Features**

Criterion	C1	C2	C3
F1	1	1	1
F3	0.524	0	0
F13	0.68	0	0
F15	0.6	0	0
F18	0.7	1	1
F19	0.167	0	0
F20	0.3	0	0.3
F21	0.227	0.159	0
F22	0.619	0.9	0.7
F23	0.085	0.633	0.25
F40	0.295	0.6	0.6
F45	1	1	1

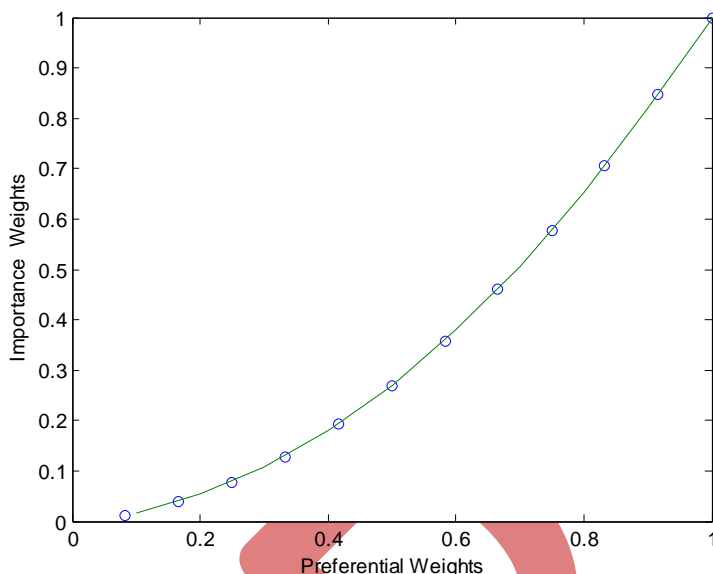
Table.14 shows the feature weights based on the relevance of the values. It reveals the data values are reliable and relevant. The weighting vector  $w$  is determined in different epochs and different criterion. For Instance, vector  $w = [0.08, 0.08, 0.08, 0.08, 0.08, 0.06, 0.06, 0.06, 0.08, 0.08, 0.08, 0.08]$  indicates that the highest values are important.

Based on the preferential and importance of the features the weights are derived. These weights are obtained from two weight vectors such as  $P$  and  $w$ . For the diffident approach, the weights are derived from interpolation function. The preferential and importance weights are normalized which are presented in table 15.

**Table 15. Normalized Vector Values of Importance and Preferential Weights**

Importance Weight	Preferential Weight
0.083	0.012
0.166	0.038
0.25	0.076
0.333	0.128
0.416	0.192
0.5	0.269
0.583	0.358
0.666	0.461
0.75	0.576
0.833	0.705
0.916	0.846
1	1





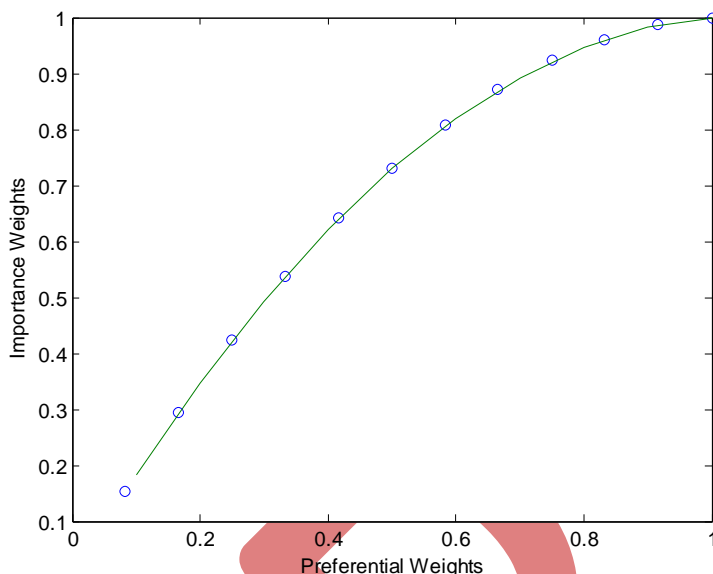
**Figure 11. Interpolation function w\* diffident approach**

Figure 11 shows the normalized weights of both preferential and importance weights of the Sonar dataset. Some features are irrelevant features in Sonar dataset, which affect the accuracy of clustering process.

In confident approach, the highly relevant and reliable feature weights are normalized for obtaining the derived weights using interpolating function.

**Table 16. Normalized Vector Values ( Confident Approach)**

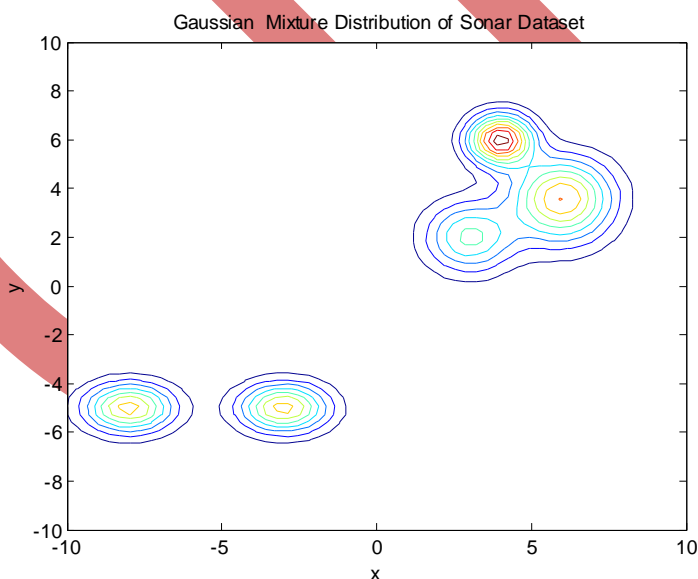
Importance Weight	Preferential Weight
0.083	0.153
0.166	0.294
0.25	0.423
0.333	0.538
0.416	0.641
0.5	0.730
0.583	0.807
0.666	0.871
0.75	0.923
0.833	0.961
0.916	0.987
1	1



**Figure 12. Interpolation function w\*confident approach**

The preferential and importance weights are used to draw the interpolating function by applying confident approach. Figure 12 shows that the normalized weights of both preferential (x-axis) and importance (y-axis) weights are diagrammatically represented as interpolation function. It divulges that the highly relevant features are used for clustering and it improves the clustering accuracy.

In several epochs, various features are selected and their weights are ordered using threshold. The Winning Component Mixtures occur in the learning process and error rate index values are calculated.



**Figure 13. Gaussian Mixture Components of Sonar Dataset**

In Figure 13, the data points are composed of mixture with independent variables and are sampled from a standard normal distribution. Various component mixtures and cluster structures are formed with the help of data points. FWOVA algorithm is used to cluster the various data points and the new winning component mixtures are obtained.

## 6. Findings

In this analysis, the highly reliable data points and relevant features are determined by using this aggregation. Here the highest weighted features in the weight vector ( $p'$ ) are considered as important ones compared to the lowest values. The highly reliable values ( $w'$ ) are considered as relevant ones for feature selection. Here, the different weights are calculated by using orness criterion. This analysis reveals the various weights and their error rate index of both model and sampling. The various experiment results are tabulated in table 17. The accuracy of the proposed approach is evaluated in the form of error rate. The proposed approach has been evaluated by dividing the dataset as training and test set. The

mean and standard deviation are the measures used for finding the accuracy of the cluster similarity. It reveals that some more irrelevant features are removed and thereby the accuracy is improved using FWOVA

**Table 17 Accuracy of test sets for each algorithms**

. Dataset	Method	Model Order (Mean ±Std)	Error Rate (Mean ±Std)
Wine d=13 N=178 K*=3	IRFS-RPEM	4.7±1.7	0.0492±0.0182
	IRRFs-RPEM	3.1±0.5	0.0509±0.0248
	FW-RPEM	2.9±0.4	0.0424±0.0234
	<b>FWOWA-RPEM</b>	<b>2.8±0.25</b>	<b>0.0418±0.0220</b>
Ionosphere d=32 N=351 K*=2	IRFS-RPEM	2.0±0.8	0.2921±0.0453
	IRRFs-RPEM	2.5±0.5	0.2121±0.0273
	FW-RPEM	2.4±0.5	0.2100±0.0260
	<b>FWOWA-RPEM</b>	<b>2.3±0.4</b>	<b>0.2095±0.0245</b>
wdbc d=30 N=569 K*=2	IRFS-RPEM	2.3±0.4	1.021±0.0546
	IRRFs-RPEM	Fixed at 2	0.0897±0.0308
	FW-RPEM	Fixed at 2	0.0776±0.0268
	<b>FWOWA-RPEM</b>	<b>1.9±0.3</b>	<b>0.0766±0.0250</b>
Sonar d=60 N=1000 K*=2	IRFS-RPEM	2.8±0.6	3.625±0.0394
	IRRFs-RPEM	2.7±0.7	0.3221±0.0333
	FW-RPEM	2.6±0.6	0.3120±0.0320
	<b>FWOWA-RPEM</b>	<b>2.5±0.55</b>	<b>0.3112±0.0313</b>

This experiment reveals that the error rate is reduced by using the proposed approach. The various clustered data points are formed using class labels in the dataset.

**Table 18. Proportions of the Average Selected Features**

Dataset	FOWA (%)	FWOWA (%)
Wine	59.60	57.45
Ionosphere	32	30.65
Wdbc	48.34	46.12
Sonar	53.36	51.26

Table 18 shows the proportions of the selected features by FOWA and FWOVA in the real world dataset. For the Wine and Ionosphere datasets, the relevant and reliable features are selected and their clustering accuracies are high. In this context, for the wine and Ionosphere datasets there is an improvement in clustering accuracy by applying the importance weights. In Wdbc and Sonar dataset, the clustering accuracy is improved when compared to FOWA. FWOVA finds the relevant and reliable features in the dataset and its predictability power is elevated, when compared to FOWA. FWOVA has better predictive accuracy with a mean of 1.95% when compared to FOWA.

**7 Conclusion**

The proposed feature reduction approach determines the irrelevant features by using WOWA. The orness criterion measures feature weights by using preferential and importance weight, which is ordered and the lower level weights are discarded. The strength of using FWOVA algorithm is that the features relevance and reliability are determined. This proposed approach estimates the parameters and obtains the highly weighted features and the error rate is reduced compared to previous approach. It reveals that the cluster similarity of the data point is improved using this approach, the selected features' relevance is strong and that the accuracy of the model is high. With the WOWA aggregation method for feature reduction, relevance and reliability information are obtained, which improves the clustering accuracy for feature

recognition. The irrelevant features are removed from the feature space and it leads to avoid overfitting. Experiments have shown the efficiency of the FWOVA algorithm in comparison with the FOWA algorithm on real-world data sets.

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