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Elasto-Plastic Analysis of 3D Frames with Generalized Yield Function

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ABSTRACT

Three dimensional three node elasto-plastic finite element has been presented. Hinges have been assumed to form at the points of integration (Gauss points) which are distributed over the length of the element. One integration point at the center and the other two near the ends. The inelastic interaction between biaxial bending moment, torque and axial force has been considered by means of generalized yield interaction surface and a flow rule with strain hardening has been associated. The element is more effective where the location of hinges is not known in advance. The concept has been applied successfully on three dimensional steel and reinforced concrete frames.

Indexing terms/Keywords

Elaso-plastic; frames; yield function

Academic Discipline And Sub-Disciplines

Engineering, Civil Engineering, Structural Engineering

SUBJECT CLASSIFICATION

Structural Engineering, Structural mechanics

TYPE (METHOD/APPROACH)

Incremental and iterative solution of material nonlinear problem

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INTRODUCTION

A number of two dimensional beam bending elements based on plastic hinge concepts have been described in the literature. For a perfectly plastic hinge (no strain hardening) the theory is trivial to formulate. For cases with strain hardening, the available models are generally of either the parallel type[Clough et al. (1965) and Porter and Powell (1971)] or the series type (Chen and Powell (1982), Giberson (1967), Litton (1975), Thom (1983) and Powell and Chen (1986)]. The second order nonlinear elasto-plastic analysis of space frames given by Ramchandra et al. (1990) includes effects of both material and geometric nonlinearities. The stress strain relationship has been assumed to be linearly elastic perfectly plastic. Riva and Cohn (1990) explored the potential of various lumped plasticity models for inelastic analysis of reinforced concrete frames and pre stressed concrete frame. Singh (1995) presented 3-D three node elastoplastic element with plastic hinges at the points of integration distributed along the length of the element.

ELEMENT CONCEPT

Three dimensional three node elasto-plastic finite element has been presented as shown in Fig. 1. Hinges have been assumed to form at the points of integration (Gauss points) which are distributed over the length of the element, one at the center and other two near the ends. The inelastic interaction between biaxial bending moment, torque and axial force has been considered by means of generalized yield interaction surface and a flow rule with strain hardening has been associated. The element is more effective where the location of hinges is not known in advance.

Element Formulation: The frame element used has six degrees of freedom per node [Singh (1995)]. The displacement vector is

$$\delta = [\, u \quad v \quad w \quad \theta_x \theta_y \theta_z \,]^T \qquad (1)$$

The strain vector is expressed as

$$\varepsilon = \left[\, \varepsilon_{xx} \quad \phi_{xy} \quad \phi_{xz} \quad \alpha \quad k_{xz} \quad k_{xy} \,\right]^T \qquad (2)$$

where

$$\varepsilon_{xx} = \frac{du}{dx}$$
$$\phi_{xy} = \frac{dv}{dx} - \theta_z$$
$$\phi_{xz} = \frac{dw}{dx} + \theta_y$$
$$\alpha = \frac{d\theta_x}{dx} \qquad (3)$$
$$k_{xz} = \frac{d\theta_y}{dx}$$
$$k_{xy} = \frac{d\theta_z}{dx}$$

The stiffness matrix of 3-D frame element is expressed as:

$$K = K_a^e + K_s^e + K_t^e + K_b^e \qquad (4)$$

where

$$K_a^e = \int B^{aT} D^a B^a \, dx$$

$$K_s^e = \int B^{sT} D^s B^s \, dx$$

$$K_t^e = \int B^{tT} D^t B^t dx$$

$$K_b^e = \int B^{bT} D^b B^b dx \qquad (5)$$

The material moduli matrices are defined as:

$$D^a = EA \qquad (6)$$

$$D^s = \begin{bmatrix} S_{xy} & 0 \\ 0 & S_{xz} \end{bmatrix} \quad (7)$$

where $S_{xy} = S_{XZ} = GA_S$ and $A_S = A/1.2$ (for rectangular section)

$$D^t = GI_{XX} \quad (8)$$

$$D^b = \begin{bmatrix} EI_{yy} & 0 \\ 0 & EI_{zz} \end{bmatrix} \quad (9)$$

The element stiffness matrix has been calculated using selective integration. The non-shear terms has been integrated using normal integration with three point Gauss quadrature. The shear terms are evaluated using reduced integration (two point Gauss quadrature) and are extrapolated to match with the integration of other terms.

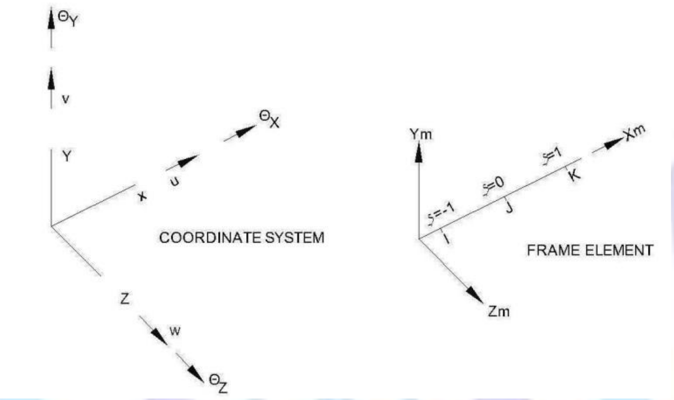

Fig. 1 Coordinate System and Frame Element

The stiffness matrix of the beam element in global coordinate system is given by

$$K_{-}^{e} = T^{T} K^{e} T \quad (10)$$

where **T** is a diagonal transformation matrix of size equal to the size of the element stiffness matrix.

INELASTIC ANALYSIS

Inelastic behavior of the element is assummed to be governed by the axial force, two flexural moments and torsional moment at a section. The section model has been assummed which is computationally efficient. Chen and Powell (1982), proposed five yield (interaction) surfaces. The surfaces differ, however, mainly in the manner in which the axial force interacts with three moments. Powell and Chen (1986) have shown that the yield surface given by:

$$F=[(M_X/M_{XU})^2 + (M_y/M_{yu})^2 + (M_Z/Mzu)^2]^{1/2} + [F_X/F_{XU}]^n \quad (11)$$

gives acceptable results in a wide range of practical domain. The exponent n is of the order of 2. Further Powell and Chen (1986) have shown that with n=1.6 the predicted behavior is satisfactory for practical purposes for simple steel structure. Singh (1995) has demonstrated the effectiveness of this yield creteria (with n=1.6) for both steel and reinforced concrete structures. It is also established through the test structures given in this paper.

The yield criterion determines the stress level at which plastic deformation begins and is written in general form

$$f(\sigma) = k(\kappa) \quad (12)$$

where σ is the stress vector, κ is the hardening parameter which governs the expansion of the yield surface.

The Eq. (12) can be written as follows:

$$F(\sigma I) = f(\sigma) - k(\kappa) \quad (13)$$

By differentiating

$$^T dF = \frac{\partial F}{\partial \sigma} d\sigma + \frac{\partial F}{\partial \kappa} d\kappa = 0 \quad (14)$$

or

adσ-Adλ = 0 (15)

where $\mathbf{a}^T = \frac{\partial \mathbf{F}}{\partial \sigma}$ and $A = -\frac{1}{d\lambda}\frac{\partial F}{\partial \kappa}\partial \kappa$ (16)

The complete elasto-plastic incremental stress-strain relationship can be written as [Owen and Hinton (1980)].

dσ = d_{ep} (17)

$$D_{ep} = D - \frac{D.D^T}{A + d_D^T a}$$ (18)

With

$d_D = D_a$

and

$A = H^{'} = \frac{d\sigma}{d\varepsilon_p} =$ tangent to the effective stress-plastic strain curve and is a function of accumulated effective plastic strain ε_p.

In the present study, the yield moments and axial forces for the reinforced concrete section have been calculated from the appropriate charts given in SP:16 (1980).

Hinges have been assumed to form at the points of integration which are distributed over the length of the element. One Gauss point is in the center of the element and other two near the ends. In framed structures particularly reinforced concrete framed structures, the frame elements are stiffer near the ends due to joint stiffnesses. So it is appropriate to assume the formation of hinges near the ends of the elements [Singh(1995)].

EXAMPLES

Both steel and reinforced concrete structures have been analyzed to study the effectiveness of the proposed model.

Steel Structures

Two steel test structures, a tubular strut and a steel space frame have been analyzed here.

Test Structure 1 - A tubular Strut

A tubular strut shown in Fig. 2 subjected to axial force and bending moment [Powell and Chen (1986)] has been studied. The geometry and the properties are shown in the same figure. The different load cases considered are: (a) Pure Bending, (b) Axial force equal to 50 percent of axial yield , then add pure bending, and (c) Axial force equal to 80 percent of the axial yield, then add pure bending.

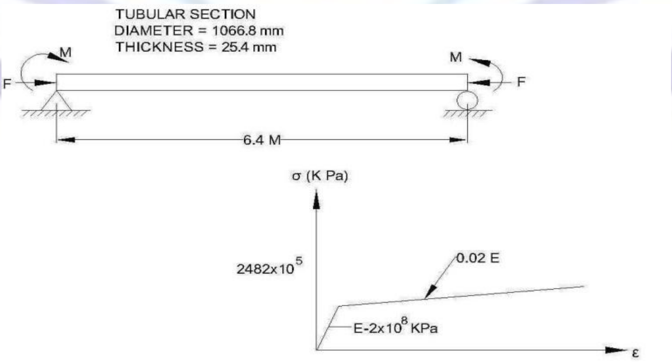

Fig.2 Test Structure 1-A Tubular Strut

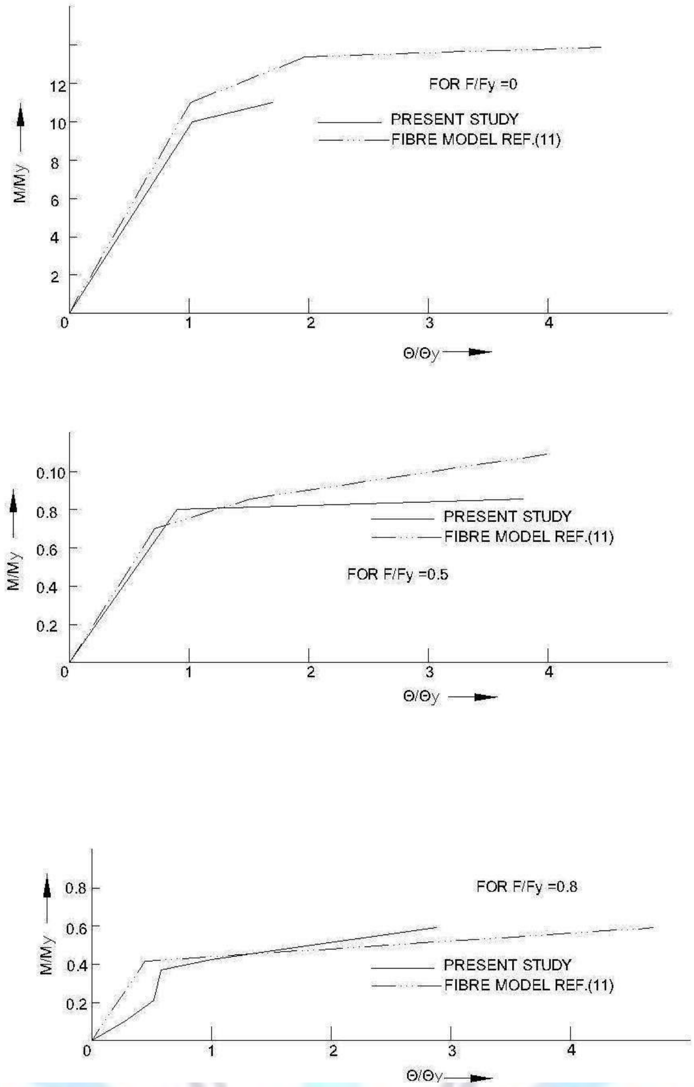

Fig. 3 Moment Rotation Relationship

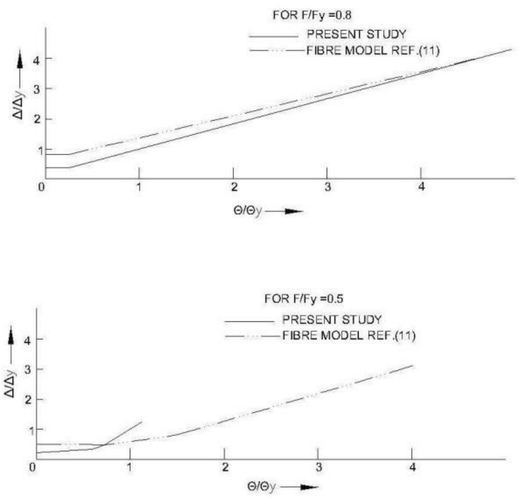

Fig. 4 Axial Deformation vs End Rotation

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The moment rotation relationships were calculated and compared with those predicted by Powell and Chen (1986). The ratios of axial extension to end rotation were also calculated and compared with those of fibre model [Powell and Chen (1986)]. The moment rotation relationships and axial deformation versus end rotation relationships are shown in Figs. 3 and 4 and have compared with those of fibre model [Powell and Chen(1986)]. A reasonably good comparison of results with those of the reported fibre model has been obtained.

Test Structure 2 - A Space frame-I

A rigid space frame consisting of four beams and four columns fixed at the base as shown in Fig. 5 has been taken for the elasto-plastic analysis [Ram Chandra et al. (1990)]. The loads shown on the frame correspond to a unit load factor. The dimensions of the frame are also shown in the same figure. The members consist of symmetrical wide flange I-sections, and values of Young' Modulus of Elasticity 203.8 kN/mm^2 and shear modulus of 78.3 kN/mm^2 have been used. The yield stress of the steel used was 351.463 N/mm^2 The members used are: 1 - 838.2mm WF at 3.575kN/m; 2, 3 and 4 - 914.4mm WF at 4.085 kN/m; 5 and 7 - 838.2mm WF at 2.918 kN/m; 6 - 762mm WF at 1.567 kN/m; and 8 - 838.2mm WF at 1.648 kN/m.

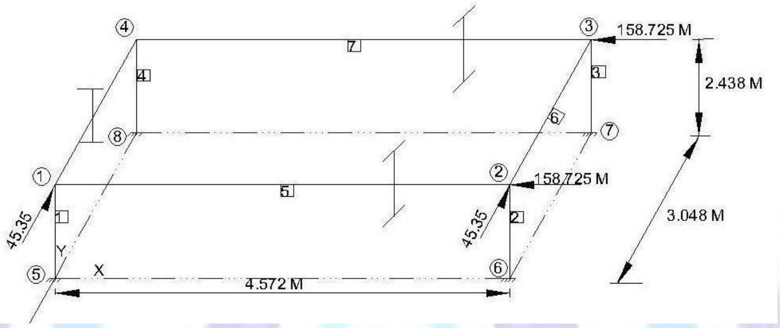
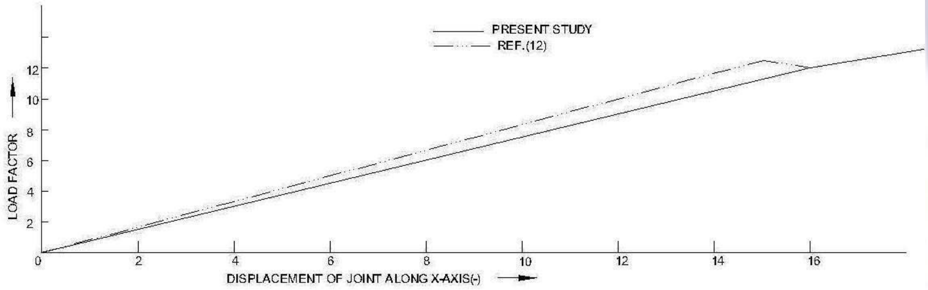
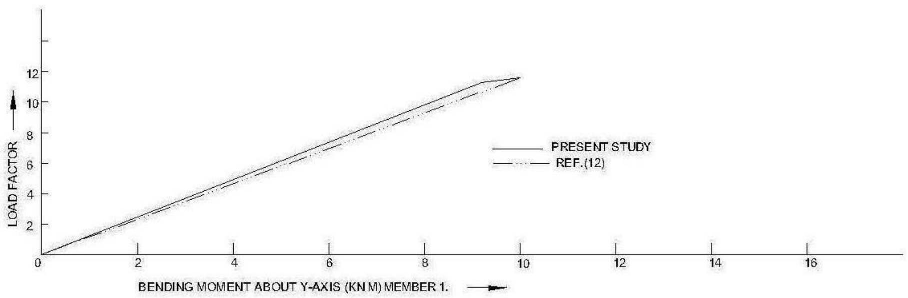


Fig. 5 Test Structure 2- A Space Frame I



Fig, 6 Load Displacement Diagram for Space Frame 1



Fig,7 Load Factors and Bending Moment in Members for Space Frame 1.

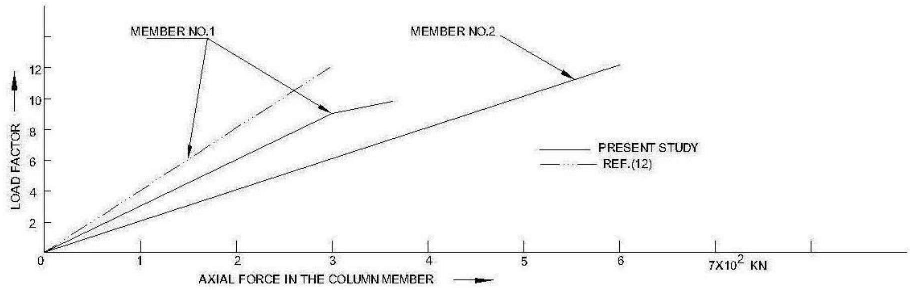


Fig. 8 Load Factors and Bending MomAxial Forces in Members for Space Frame 1.

The load displacement behavior in terms of displacement of joint 2 in X-direction has been shown in Fig. 6 and are compared with that obtained by Ram Chandra et al. (1990). Fig. 7 shows the bending moment variation at end 1 of member 1 and that obtained by Ram chandra et al. (1990). Fig. 8 shows the comparison of axial forces in the column members 1 and 2 at different load factors with those obtained by Ram Chandra et al. (1990). From the results it can be seen that axial forces in the column members are influenced by the effect of nonlinearties once the plastic hinge has been formed in the member.

The nonlinear behavior of the frame is exhibited clearly in load displacement diagram Fig.6 for the horizontal deflection of joint 2. The response of the frame becomes nonlinear after the development of first plastic hinge in the frame as shown in Fig. 5. A large number of small increments were needed to study the behavior till collapse. It has been observed that the results obtained by the proposed algorithms are in good agreement with the reported results.

Reinforced Concrete Structures

Concrete is not purely elastic material. The plastic flow (creep) has been observed in it. The modulus of elasticity varies with stress rate and magnitude of the stress. The effective reinforced concrete section also varies with the stress level. Both the modulus of elasticity and effective cross section decrease with the increase in stress level. In the 'elastic' range, either their values should be varied or an average value may be used. The reduction of the elastic rigidity EI by 50 percent has been suggested by many researches to define an average value [Anderson and Townsend (1977), Saatcioglu (1984), and Mozzami and Bertero (1987)]. In the present study, 50 percent reduction in short term value of static modulus of elasticity of concrete and effective sectional properties both calculated as per IS: 456-1978 have been assumed for the entire 'elastic' range prior to the development of ultimate yield surface.

Test Structure 3 - A portal Frame

The reinforced concrete portal frame shown in Fig. 9 (a) tested by Bertero and McGlure (1964) and analyzed by Sharma (1983) and Thanoon (1993) has been taken as test structure 1. The frame has been assumed to be fixed at base and idealized as shown in Fig. 9(b). The geometry, loads and properties are shown in the figure. The load deflection curve obtained experimentally by Bertero and McGlure (1964) and analytically by Sharma(1983) and Thanoon (1993) are compared with that obtained by using the proposed formulation in Fig. 9(d). Sharma used nonlinear moment-curvature relationships and performed numerical integration at Gauss points. However, Thanoon used lumped plasticity model with rigid ends and nonlinear stiffness relationships. The load deflection behavior and the failure load obtained by using the proposed algorithms are in reasonably good agreement with the reported experimental and the analytical results. It is observed that the results obtained by the proposed algorithms using the distributed plasticity are closer to the experimental results than those obtained by the lumped plasticity.

Test Structure 4 - A Space Frame II

The single storey one bay reinforced concrete space frame shown in Fig. 10(a) and previously analyzed by Thanoon (1993) has been chosen as test structure 2. It has been idealized by eight beam-column elements. The coordinate system, dimensions and other properties are shown in Figs 10(a), (b) and (c). A load system which include all types of stresses i.e. axial, shear, bending and torsion in the frame is considered for the study. Thanoon analyzed the structure with and without slab both by considering and neglecting torsion in its yield criteria. The frame without slab has been analyzed for the present study as it is intended to test the inelastic formulation for the frame elements only. The results are compared with those reported by Thanoon in which the slab has not been considered but the torsion in the yield criteria has been considered. Thanoon has used lumped plasticity model with rigid ends and with nonlinear stiffness relations. The comparison of results is presented in Fig. 10(d). The results obtained are in good agreement with the reported results.

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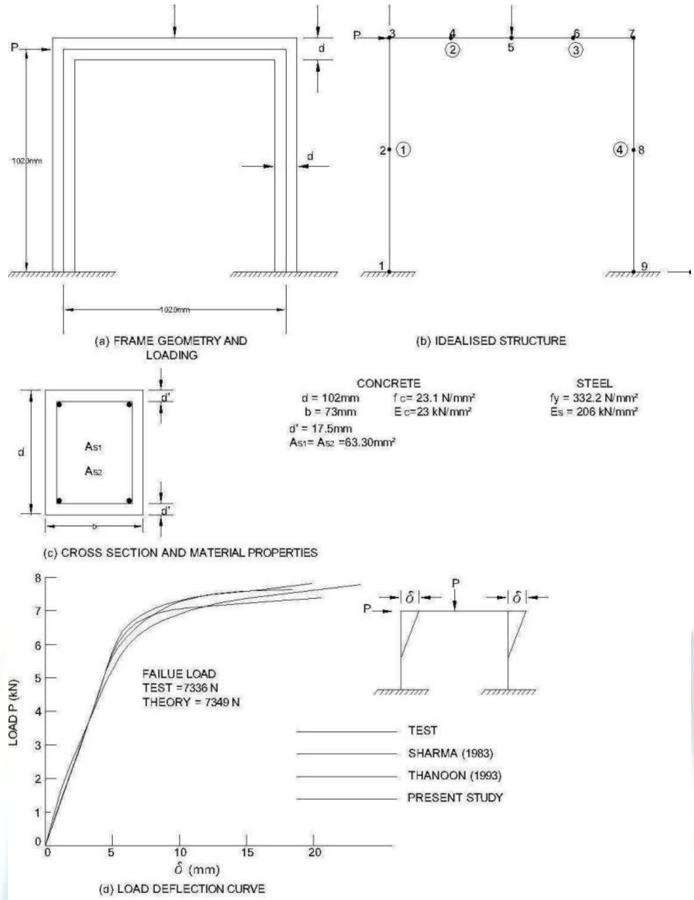

Fig. 9 Test Structure 3- A Portal frame

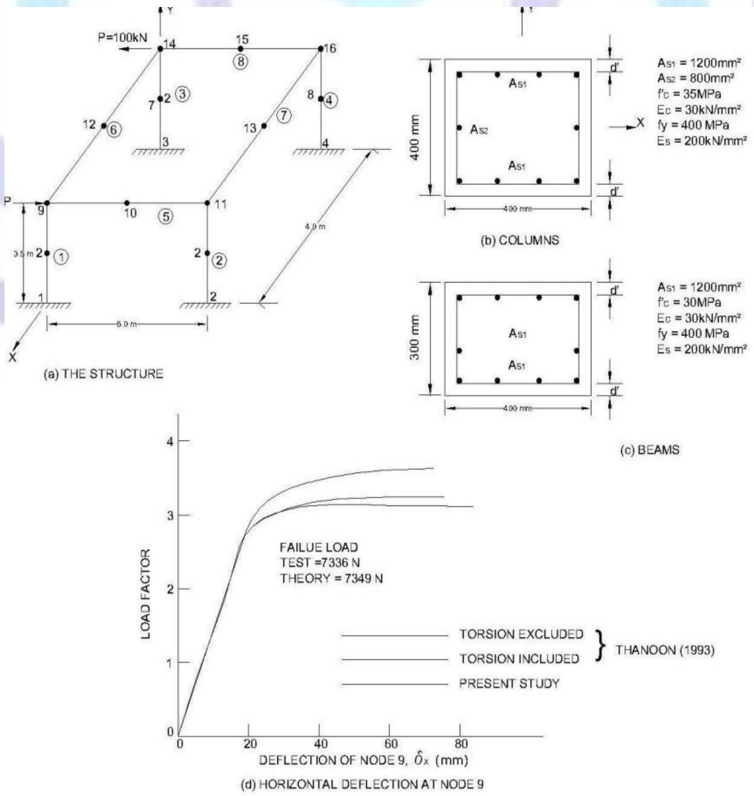

Fig. 10 Test Structure 4- A Space Frame II

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CONCLUSIONS

The proposed elasto-plastic element with plastic hinges at the Gauss points provides a simple means of modeling the inelastic response in 3-D structures particularly where the locations of hinges are not known in advance. The proposed model is able to incorporate properties like strain hardening, flow of material and generalized yield function which includes axial force, biaxial moment and torsion. The concept has been applied successfully on the 2-D and 3-D steel and reinforced concrete frames. The proposed algorithms predict the sequence of formation of the plastic hinges in the frame elements upto failure and development of mechanism. The element can give quite accurate results if appropriate care is taken in specifying the hinge properties and the selection of mesh.

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NOTATIONS

The notations used in this study are listed below. Small bold letters represent a vector and capital bold letters represent a matrix. Sometimes, a symbol may have an alternate meaning but in such a case, the context is sufficient to avoid confusion.

A	cross-sectional area.
A_S	Shear area
$\mathbf{a}$	Flow Vector
$\mathbf{B}^a$, $\mathbf{B}^s$, $\mathbf{B}^t$, $\mathbf{B}^b$	Strain-displacement matrix: axial, shear, torsional and bending respectively
$d\lambda$	A dimensionless magnitude
$\mathbf{D}^a$, $\mathbf{D}^s$, $\mathbf{D}^t$, $\mathbf{D}^b$	Material property matrix: axial, shear, torsional and bending respectively.
$\mathbf{D}$	Material property matrix
$\mathbf{D}_{ep}$	Elasto-plastic material property matrix
E	Modulus of elasticity
F_X, F_{XU}	Axial force and axial yield force respectively.
f	Yield function
G	Shear modulus
I_X, I_Y, I_Z	Moment of inertias of beam element about X, Y and Z axes respectively.
$\mathbf{K}^e$, $\underline{\mathbf{K}}^e$	Stiffness matrix of element in local and global coordinates respectively.
$\mathbf{K}_a{}^e$, $\mathbf{K}_s{}^e$, $\mathbf{K}_t{}^e$, $\mathbf{K}_b{}^e$	Axial, shear, torsional and bending stiffness matrices of beam element, respectively.
κ	Material parameter
M_X , M_Y , M_Z,	Moments for beam element about X , Y and Z axes, respectively.
M_{XU} , M_{YU} , M_{ZU}	Yield moments for beam element about X, Y and Z axes, respectively.
$\mathbf{N}$	Matrix of shape functions
n	exponent
S_{XY}, S_{XZ}	Shear stiffness coefficients for beam element.
$\mathbf{T}$	Transformation matrix.
u, v, w	Translational degrees of freedom along X, Y and Z axes.
dx	Incremental length along X axis.
$d\alpha$, $d\varepsilon$	Incremental stress and strain
α	Twist, the angle between the normal of surface and X-axis.
θ_X, θ_Y, θ_Z	Rotational degrees of freedom about X, Y, and Z axes, respectively.
δ	Displacement vector.
ε_{xx}, ϕ_{xy}, ϕ_{xz}	Axial strain and shear strains
k_{XZ}, k_{XY}	Curvatures or bending strains

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Short Term Course:Organized: 03, Attended: 24